# Kossey <br> THE HEBREW CALENDAR: <br> A Mathematical I ntroduction <br> Prepared by: JOHN A. KOSSEY Edi tor: HERMAN L. HOEH <br> FI RST EDI TI ON <br> AMBASSADOR COLLEGE PRESS <br> Pasadena, Cal i forni a 1971, 1974 Edi ti on 

PROGRAM I

## USI NG THE TI ME UNI TS OF THE HEBREW CALENDAR

## I NTRODUCTI ON

Why shoul d YOU st udy the Hebrew cal endar?
One of the maj or i dentifying si gns of the Church of God is the observance of the Sacred Festivals. As you study the twenty-third chapter of Leviticus, you will notice that God employed a cal endar to i ndi cate when each hol y day must be properly kept during the year. The Jews were gi ven the responsi bility of preserving that cal endar for the rest of the world.

Si nce it is the responsi bility of the Church to announce the time of each festival to the congregations, detailed understanding of the Hebrew cal endar is not even necessary for a lay church member. When a hol y day is to be kept is not for the indi vidual Christian to decide.

On the ot her hand, the education you are privileged to recei ve as an Ambassador College student equi ps you with a special depth of bi blical understanding. Shal low or sket chy know edge of basic background areas would lessen one's effectiveness. But working out the cal endar principles yourself is going to wi den your perspective.

You al ready know that the hol y days portray God's master pl an of sal vati on for manki nd. Shoul dn't you al so have a working know edge of the very cal endar whi ch houses God's Sacred Festival s?

Thi s i s why a study of the Hebrew cal endar is incl uded in Theol ogi cal Research I-II.

## THE PURPOSE OF THESE LEARNI NG PROGRAMS

Your study of the Hebrew cal endar in this course has tho maj or facets. One is the hi storical devel opment of the cal endar. This is the primary function of the cl ass lectures. The other is for you to achi eve needed computational facility with the cal endar itself.

Fort unatel y, the Hebrew cal endar requi res surprisingly little mat hematical sophistication. A fifth grade background in arithmetic will suffice! Neverthel ess, a certai n number of skills and concepts must be learned for you to become adept at working wi th the Hebrew cal endar. These prograns are desi gned to provi de you with that understanding and practice.

Just what will you be able to accomplish when you compl ete this series of learning programs?

For any year, such as $4 \mathrm{BC}, 31 \mathrm{AD}, 1520 \mathrm{AD}$, and 1979 AD , you will correctly det er mi ne the dates on a common Roman cal endar of the hol y days listed in Leviticus 23.

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How Iong will this oper ation require? With nothing but a pencil and a bl ank sheet of paper, you might need anywhere from thirty to forty-five minutes. If you use a table of reduced numbers (which is included in one of the prograns), it mightake you only ten to fifteen minutes.

That skill is the OVERALL GOAL of thi s series of prograns. Another less tangi ble ai mis to gi ve you the confidence that you can actually SUCCEED i $n$ working a cal endar probl ent

To that end, each learning programtakes a necessary part of the mai $n$ goal, and gi ves you the practice needed to become adept at it. Success will breed success as you progress!

The first page of each I earning program has a clear statement of what you must be doing by the time you complet the program You might thi nk of each program as a "checkpoi nt" on route to your destination. Be sure you can accomplish each programgoal before going on to the next one.

One word of caution. Have you ever learned mathematics si mply by gl ancing over the text book or wat ching someone el se work through a probl em ${ }^{\gamma}$ No, you can't! The exercises in each program are entirel y for your benefit. In most cases, these exercises will be worked out in detail later in the program This is for you to have a model with whi ch to compare your own procedures and to check your work i mredi at el y for errors.

But WORK you must! Proverbs 4: 13 says to "take fast hol d of i nstruction; l et her not go. "Learning the mathematical operations of the Hebrew cal endar will take ACTI VE EFFORT. With this diligence, you will achi eve the exhilaration only success can bring.

Cont ent s
Program
I nt r oduction
$i$
1
2
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8
Using the time units of the Hebrew cal endar
Cal culating the day of the week of the mol ad Ti shri
Cal culating the day of the month of the mol ad Tishri
Using tabl es to find the mol ad Tishri
Making Roman I eap year and Julian/ Gregorian corrections
Appl ying the post ponement rul es to find Tishri one
Counting the days of the week and the days of the month
Determining the dates of the annual festivals
$\qquad$
PERFORMANCE GOAL 1A:
W thout hesitation or uncertai nty you will write (or recite) from memory in any order the following time rel ationshi ps:

| 1 part (chal yek) | $=76$ moments (regai m) |
| :---: | :---: |
| 1 hour | = 1080 parts (chal aki m) |
| 1 day | 24 hours |
| 1 week | 7 days |
| 1 I unar month | 29 days 12 hours 793 parts |
| 1 common year | 12 I unar mont hs |
| 1 i nt ercal ary year | $=13$ l unar mont hs |
| 1 ni neteen year cycle | $=235$ I unar months, or 12 common years \& 7 intercal ary years |

PERFORMANCE GOAL 1B:
W th onl y paper and pencil (or pen), you will accurately add,

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subtract, multiply, and divide the time rel ationshi ps listed in IA as necessary to compute specified problens. Large numbers will be "reduced to lowest terms" when requested. For example,

28 hours reduces to 1 day, 4 hours
31 hours 650 parts reduces to 1 day, 7 hours, 650 parts.

## PERFORMANCE GOAL 1A

In order for you to become adept with cal endar cal cul ati ons, you must become very familiar with a mi mal number of time rel ationships. Some of these you al ready know, ot hers requi re memorization. These numbers will occur so frequently that you si mply must know them Ot herwi se, the lessons will take much longer to understand and you will feel frustrated in the process. Learn them now-- for your own benefit.

For a detailed yet conci se description of the time el ements of the Hebrew cal endar, consult either of the following ref erences:
"The J ewi sh Encycl opedi a", vol ume 3, "Cal endar" (either new or old edition).

Burnaby, Sher rard Beaumont, "El ements of the J ewi sh and Muhammadan Cal endars". London: George Bell \& Sons, 1901. 554 pages. (See chapter II, page 21.)

Here is a brief expl anation of some of these time el ements to assist your memorization.

DAY: Genesi s 1: 5 shows that the day begins in the evening. "And the evening and the morning were the first day. "Al though each day begins at sunset, 6 PM is the arbitrary commencement of a new day for CALENDAR CALCULATI ONS.

Christ put his di vine approval upon di vi ding the day into twenty-four hours. See John 11: 9, whi ch states, "Are there not twel ve hours in the day?" Cont ext shows that this verse refers to the dayl i ght portion of a twenty-four hour period. For any given day the periods of darkness and light are usually unequal. The total length of a day, however, is al ways 24 hours--except for Di vi ne intervention!

HOUR: Instead of being di vi ded into minutes and seconds, the hour is di vi ded into parts, or chal akim One hour consists of 1080 parts, or 3600 seconds. Using parts instead of minutes and seconds has the advantage of el im nating fractions. (The smaller unit, the moment or rega, is sel dom needed for fundamental calcul ations.) Both the hour and the part are considered fixed units anywhere on earth, just as the mi nute and the second are non-varying time el ements.

MONTH: A l unar month is the time needed for the moon to revol ve ar ound the earth. Even though this period varies from month to month, 29 days 12 hours 793 parts is the traditional average used for cal cul ation. Act ual cal endars cannot be based upon 29 1/2 days, so the Hebrew cal endar i ncor porates mont hs of 29 and 30 days.

YEAR: The Hebrew cal endar has two basic types of years, common and i ntercal ary. (The latter termis al so called "entoolismic.") An int ercal ary Hebrew year will have 30 additional days, so it can al so be called a l eap year. By contrast, recall that the Roman leap year has 366 days instead of 365 . You should al so remenber that the foll owing terms are interchangeable for the Hebrew cal endar:

LEAP (year or mont h) = embol ismic (year or month) = I NTERCALARY( year or mont h)

Common years will have 353, 354, or 355 days. Leap years may have 383, 384, or 385 days.

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19 YEAR CYCLE: The Western world is accustomed to a sol ar year of 365 1/4 days, si nce the Roman cal endar in common use is sol ar. Thi s means that a gi ven month of the year will al ways occur during the same season. January, for example, is i nvariably a wi nter month.

On the ot her hand, the Hebrew year by itself does not closely match the length of a sol ar year. Twel ve mont hs whi ch are each approxi mately 29 1/ 2 days results in a year whi ch has onl y 354 days -about el even days less than a sol ar year of $3651 / 4$ days. A common Hebrew year is thus SHORTER than a Roman year.

What about a Hebrew I eap year? Thi rteen months of $291 / 2$ days is 383 1/ 2 days, whi ch is LONGER than a sol ar year.

God has or dai ned that the hol y days must be kept "in thei r seasons" (Lev. 23: 4). He al so appointed BOTH the sun and the moon "for si gns, and for seasons, and for days, and years" (Genesis 1:14). Thi s means that the cal endar whi ch God desi gned to house His sacred festivals would be luni-sol ar. The months must occur at the proper times for the holy days to fall within the proper season of the year.

How, then, are the Hebrew I unar mont hs rel at ed to the sol ar year? Every 19 sol ar years (of 365 1/4 days), the mon revolves around the earth 235 times, each "I unation" being on the aver age 29 days, 12 hours, 793 parts. Thi s remarkable astronomical rel at i onship makes it possi ble to combi ne commn years and leap years toget her within a fundamental pattern that repeats itself every ni neteen years:

```
12 common years ( }12\mathrm{ mont hs each) is }144\mathrm{ months (each month is
    7 leap years (13 mont hs each) is 91 months 29d, 12 h,
79---------------------------------793p)
19 Hebrew years is 235 months = 19 sol ar years
```

You shoul d be aware, however, that 235 I unar months is about an hour and a half less than 19 Jul ian years. (To be preci se, 235 I unations is 1 hour, 485 parts LESS than 19 Jul ian years.)

The 19-year cycle is al so known as the cycle of Meton, or the Met oni c cycle.

Be sure you have MEMORI ZED the time rel ationshi ps il sted at the begi nni ng of thi s section! Don't assume that you know them "pretty well."

## PERFORMANCE GOAL 1B

The source of many er rors in cal endar cal cul ations is faulty arithmetic. Mbst occur in the basic addition, subtraction, and multiplication operations. All of these skilils you learned bef ore high school. Consequently, some revi ew and practice are absol utely necessary. WORK the probl ens; don't be del uded into thi nki ng that glancing over them will suffice.

ADDI TI ON: The key to successful computation rests in one simple rul

> add onl y likes toget her

Just as 3 apples and 4 or anges don't equal 7 apples, nei ther does adding 11 hours to 4 days equal 15 hours. Never put a number down without bei ng positively certain that you know what it represents. The best way is to label every number: 671 p is 671 parts. Another way is to keep li ke quantities in clearly defi ned col ums:

| days | hours | parts |
| :---: | :---: | :---: |
| -----12 | ---103 |  |
| 1 | 12 | 1186 |

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$\begin{array}{lll}\text { Thi s is the same as: } & \begin{array}{l}12 \mathrm{~d} \\ 3 \mathrm{~d}\end{array} & 12 \mathrm{~h} \\ & 15 \mathrm{~h} & 103 \mathrm{p} \\ & 186 \mathrm{p}\end{array}$
Sel ect a format you like and use it consi stently. Here are some sample probl ens:

| 5 d | 15 h | 175 p |
| :---: | :---: | :---: |
| + 1 d | 7 h | 801 p |
| 6 d | 22 h | 976 p |

So I ong as you add onl y like quantities, no outstanding difficulties will happen.


The numbers can become very large, as you can see. Later on you will I earn how to reduce these to lowest terms.


Adding a whol e string of numbers toget her will of ten occur. Don't be afraid to double and triple-check your arithmetic!

Sol ve the foll owing probl ens bel ow. If you aren't "comf ortable" by the time you complet them try to make up a few of your own for additional practice.

| 1. 1) |  | 6 d | 17 |  | 879 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $+$ | 3 d | 14 |  | 198 |
| 1. 2) |  | 67 d | 95 | h | 777 |
|  | $+$ | 275 d | 777 | h | 2589 |

1. 3) 

|  | 55 d | 178 h | 976 p |
| ---: | ---: | ---: | ---: |
| 31 d | 1 h | 134 p |  |
| 178 d | 23 h | 1937 p |  |
| 225 d | 9 h | 11581 p |  |
| 308 d | 768 h | 649 p |  |
| $+\quad 12 \mathrm{~h}$ | 793 p |  |  |

1.4)

| 131 d | 29 h | 433 p |  |
| ---: | ---: | ---: | ---: |
| 227 d | 8 h | 191 p |  |
| 0 d | 1 h | 485 p |  |
| + | 130 d | 12 h | 883 p |

The answers to these probl ens appears bel ow.

| 1. 1) | 9 d | 31 h |
| :--- | :---: | ---: |
| 1. 2) | 342 d | 872 h |


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| :--- | ---: | ---: | ---: |
| 1. 3) | 826 | $d$ | 991 |
| 1. 4) | 488 | d | 50 h |

REDUCTI ON: Bef ore attempting to subtract or multiply these numbers, it will be hel pf ul for you to understand how they are reduced.

You have learned how to reduce quantities such as inches to feet and yards, or seconds to hours and minutes, back in $j$ uni or hi gh mat hemat i cs:

| 45 | i nches | reduces | to |  | 1 | yard |  | f eet | 9 | i nches |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 49 | i nches | reduces | to |  | 1 | yard | 1 | f oot | 1 | i nch |
| 436 | seconds |  |  | " | 0 | hours | 7 | mi . | 16 | seconds |
| 3600 | seconds | " |  |  | 1 | hour |  | min. | 0 | seconds |

Reduction of I arge numbers is not difficult. An EQUIVALENT way of saying exactly the same thing is all that you are doing. Both quantities are equal.

The primary units in your cal cul ations will be days, hours, and parts. Each of these quantities are rel ated to each ot her, as you have already l earned from performance goal 1A.

1. 5) Compl et e the foll owing:
_____-_ parts $=1$ hour
_____-_ hours = 1 day
The answers, of course, are 1080 parts in an hour, and 24 hours in a day. This means that 1080 parts can be written as:

$$
0 \text { days } 1 \text { hour } 0 \text { parts }
$$

In like manner, 24 hours can be shifted to the days col um:
1 day
0 hours
0 parts.

Take a quantity like $3 \mathrm{~d}, 49 \mathrm{~h}, 1400 \mathrm{p}$. How do you reduce it? What you want to do is transfer all EXCESS WHOLE HOURS contai ned in the parts col um to the hours col um. Then you will take out all the WHOLE days cont ai ned in the hours col um and move them to the days col um. J ust like reading Hebrew, you work from right to left!

To reduce numbers, apply the following oper at ions:

1) Di vi de the parts in the parts col umm by 1080. The whole number in the quotient represents hours.
2) Add the whol e number in the quotient to the hours col um. Thi s becomes the "revi sed" hours. Pl ace the remai nder in the "reduced" parts col um. If remai nder is 0 , put this number in that col um.
3) Take the revi sed number of hours and di vi de this by 24 . The whol e number in the quotient is the number of whol e days.
4) Add the whol e number of days to the days col um, and pl ace the remai nder in the "reduced" hours col um.

Reduced numbers for days, hours, parts will have:
any number of days 23 or less hours 1079 or less parts
Here's how to apply these rules to reduce $3 \mathrm{~d}, 49 \mathrm{~h}, 1400 \mathrm{p}$ :
1)
1 h
1 h 320 p
1080 / 1400 p
2)

$$
\begin{aligned}
& 1 \mathrm{~h} \quad 320 \mathrm{p} \\
& +3 \mathrm{~d} 49 \mathrm{~h} \\
& -3 \mathrm{~d} 50 \mathrm{~h} 320 \mathrm{p}
\end{aligned}
$$

3) 


4) $\quad 2 d 2 h 320 \mathrm{p}$

$$
\begin{aligned}
& +3 \mathrm{~d} \\
& ---5 \mathrm{~d} \quad 2 \mathrm{~h} 320 \mathrm{p}
\end{aligned}
$$

REDUCED
Probl ens with large numbers are handl ed the same way. The answer to 1. 2 is 342 d 872 h 3366 p.
1)

3 h 1080 / 3366 p

3240
126
2)

$$
\begin{array}{r}
3 \mathrm{~h} \quad 126 \mathrm{p} \\
+342 \mathrm{~d} 872 \mathrm{~h} \\
-342 \mathrm{~d} \quad 875 \mathrm{~h} 126 \mathrm{p}
\end{array}
$$

3) 
4) $\begin{array}{r}36 d \quad 11 \mathrm{~h} 126 \mathrm{p} \\ +342 d\end{array}$

378 d 11 h 126 p

Reducing numbers isn't difficult. Your accuracy will be enhanced if you consi stently stick to a single format. Sl opping numbers down haphazardly on the page is inviting computational errors.

1. 6) Reduce 5, 796 parts.
1. 7) Reduce 579, 600 parts.
1. 8) Reduce 85 d 91 h 150, 000 p.
1.9) Reduce 567 d 5228 h 254, 404 p.

The answers to 1.6 - 1.9 are on the next pages. Wbrk these probl ems on
separ ate paper, then compare your cal cul ations with the compl et e arithmetical details supplied.

1. 6

| 1080 / | 5796 |
| :---: | :---: |
|  | 5400 |

396
0 d 5 h 396 p answer

1. 7
536 h
536 h 720 p; 536 hours must be reduced.
 5400
-------
3240
7200
6480
--720
22 d 22 d 8 h 720 p answer $24 / 536$ h 48
56
48
8
2. $8 \quad 85 \mathrm{~d} 91 \mathrm{~h} 150,000 \mathrm{p}$


(3)
9 d
24 / 229 h
216
13 h
9 d 13 h ; add toget her the reduced 960 p , 9 d 13 h , and 85 d.
(4) 13 h 960 p
85 d
94 d 13 h 960 p $\quad$ answer
3. $9 \quad 567$ d 5228 h 254, 404 p
(1)

235 h
1080 / 254404 p 2160
---980 3240 6004 5400

604 p
(2) $567 \mathrm{~d} \quad 5225 \mathrm{~h} \quad 604 \mathrm{p}$

567 d 5463 h 604 p
(3) 227 d
24 / 5463 h 48

66
48
183
168
15 h
(4) $227 \mathrm{~d} \quad 15 \mathrm{~h} \quad 604 \mathrm{p}$ 567 d

794 d 15 h 604 p

MUTI PLI CATI ON: After adding days, hours, and parts, you will find that multipl ication is quite easy. The i mportant key to correct multipl ication is that ALL TERMS must be multiplied! Forgetting to multiply days while you complete the operation for the hours and parts can easily happen if you aren't wary.

Look at a sample problem
2 d 16 h 595 p $\times 3$
6 d 48 h 1785 p Notice that the multiplier oper ates on the days, the hours, and the parts.

Multipl ication can hel p you sol ve many problens that woul d take much l onger by strai ght addition. Here is an example:

How many days, hours, parts are in an AVERAGE common year?
You should easily remenber:
The number of I unar mont hs in a common year;
The number of days hours and parts in a lunar month.
If you cannot recall these numbers, return imedi at el y to the first page of this section and drill intensel $y$ on the time rel ations listed in 1A.

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A common year has 12 months. Each month has 29 d, 12 h, 793 p. Therefore an average common year has:

| 29 d | $12 \mathrm{~h} \quad 793 \mathrm{p}$ |
| :---: | :---: | :---: |
| $\times 12 \mathrm{month}$ |  |

If this number were to be used in a series of computations, it could stand "as is." On the other hand, when it is the final answer, can't you see the need to reduce such a number? You may want to verify that an aver age common year has 354 d 8 h 876 p . (This is worth remenber ing. )

1. 10 How many days, hours, and parts are in an average intercal ary year?
2. 11 What number of $d, h, p$ are in the 19 year cycle?
3. $127 \times(5 \mathrm{~d} 21 \mathrm{~h} 589 \mathrm{p})=$
4. 13 ( 4 d 8 h 876 p$) \times 12=$ $\qquad$
5. 14 ( 18 d 15 h 589 p$) \times 6=$ $\qquad$

Problems 1. 10-1.14 are worked out for you bel ow.

1. $1029 \mathrm{~d} 12 \mathrm{~h} \quad 793 \mathrm{p}$ x 13 mont hs

| 87 | 36 | 2,379 |
| :--- | :--- | :--- |
| 29 | 12 | 793 |

$377 \mathrm{~d} 156 \mathrm{~h} 10,309 \mathrm{p}$ This reduces to 383 d 21 h 589 p.

1. 11 You may cal cul ate this probl em by t wo methods:
a) add the l engt hs of 12 common years and 7 leap years toget her, using the exampl e and 3. 10.
b) Find the I ength of 235 I unations.

Here's how met hod b) is sol ved:


Thi s reduces to
41 d 6 h 883 p .

1. $13 \quad 4 \mathrm{~d} \quad 8 \mathrm{~h} \quad 876 \mathrm{p}$

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$\times 12$

| 8 | 16 | 1752 |  |
| :---: | :---: | :---: | :---: |
| 4 | 8 | 876 |  |
| -48 | $d$ | 96 | h |

Thi s reduces to 52 d 9 h 792 p.

1. 1418 d $15 \mathrm{~h} \quad 589 \mathrm{p}$ $\times 6$

108 d 90 h 3534 p
Thi s reduces to 111 d 21 h 294 p.

SUBTRACTI ON: Thi s is the most error-prone operation for many students. Two facets of subtraction are particularly troubl esome:
a) bor rowing
b) negative numbers

First look at a strai ght-forward subtraction problem


Notice that the minus sign affects all the quantities in the second line. Obvi ously, adding one of the units and subtracting the others isn't kosher! The first principle to keep in min:

SUBTRACT ALL TERMS called for.
Practice on two of these problens, comparing your answer with that gi ven.

1. $15-\quad \begin{array}{llllll}6 & d & 6 & h & 883 & p \\ & - & d & 5 & h & 785\end{array}$
answer: 5 d 1 h 98 p
2. 16 - ( $74 \mathrm{~d} \quad 14 \mathrm{~h} \quad 45 \mathrm{p})$ 111 d 21 h 294 p
answer: 37 d 7 h 249 p (You CAN subtract "upside down.")

In the next example, "bor rowing" will be necessary:
8 d 23 h 403 p

- $\quad(6 \mathrm{~d} \quad 22 \mathrm{~h} 528 \mathrm{p})$

Si nce 1080 parts are in an hour, 8 d 23 h 403 p can be changed to $8 \mathrm{~d}(23-1) \mathrm{h}(403+1080) \mathrm{p}$, or 8 d 22 h 1483 p . Be sure you understand that nei ther expressi on is different. Borrowing merely pl aces the number in a more conveni ent formfor subtraction.

By borrowing quantities as needed, the problem becomes ordi nary subtraction:

$2 \mathrm{~d} \quad 0 \mathrm{~h} 955 \mathrm{p}$
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I n order to firmup the concept of bor rowing in your mind, examine another ill ustration:

$$
\begin{array}{r}
3 \mathrm{~d} \\
-\quad(1 \mathrm{~d} \quad 18 \mathrm{~h} \quad 6000 \mathrm{p}) \\
\hline
\end{array}
$$

On this probl em you cannot simply take one hour and transfer 1080 parts. A quick estimate will tell you that at least 6 hours need to be changed to parts. Only 3 are in the hours col um. Where do you get them

First, make up for the lack of hours in the second col um by converting a day (or more!) into hours. 3 d 3 h 0 p transfers to
$2 \mathrm{~d}(24+3) \mathrm{h} 0$ parts, or 2 d 27 h 0 p.
Next borrow 6 hours and change themto parts:
$\begin{array}{lrrr}2 & \mathrm{~d} & (27-6) & \mathrm{h} \\ 2 \mathrm{~d} & 21 & (6480+0) & \mathrm{p} \\ \mathrm{d} & \text { or },\end{array}$
Finally, subtract as requi red by the original problem


Once in a while, you will find that in borrowing parts, you loose too many hours for subtracting the hours. (I nstead of 18 hours, suppose you needed to subtract at l east 22 hours in the above example.) When that happens, si mply bor row agai $n$ from the next col um to the left.

Bor rowing is a reverse of reduction. Remenber that both quantities are equi val ent -- bef ore reduct i on and after reduction, or bef ore and after bor rowing. A "gol den rule" for these oper ations:

1. $\left.17-\begin{array}{rlllrl}40 & d & 6 & h & 104 & p \\ & -(35 & d & 2 & h & 1141\end{array}\right)$
answer: 5 d 3 h 43 p
2. 18 - $\begin{array}{r}136 \\ \text { d } \\ 89 \\ d\end{array} \quad 123 \mathrm{~h} \quad 5291 \mathrm{p}$
answer: 47 d 91 h 852 p
3. $19 \quad 21 \mathrm{~d} \quad 30 \mathrm{~h} \quad 811 \mathrm{p}$

- ( 17 d 69 h 2050 p)
answer: 2 d 7 h 921 p

The last complication to hurdle with subtraction is negative numbers. By borrowing, you were able to keep numbers positive. But sometimes the arithmetic is si mplified by allowing numbers to become negative-without bor rowing.

There is nothing mysterious about negative time. All that it means is

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ti me BEFORE a prescribed reference poi nt. Such a reference poi nt is the bl ast-off of a mon launch. Events before lift-off are considered negat ive times, as you heard on many tel ecasts: "t M NUS ei ght mi nut es and hol ding. "After the rocket lifts off, time becomes positive -- with reference to the ZERO Iift-off.

An expressi on like days, hours, parts refers to a particul ar instant in time. When you are computing a problem some of these numbers will occasi onally become negative. Nonet hel ess, the expressi on is still i dentifying a preci se moment. For example look at:

$$
5 \mathrm{~d}-2 \mathrm{~h}-810 \mathrm{p}
$$

Does thi s specified time occur on the fifth day or on the fourth? The answer is the fourth day. This is immedi at el y clear if you change the expressi on to all positive signs by "borrowing. "These are the steps:


So 5 days -2 hours -810 parts is the same as 4 days 21 hours 207 parts.

You shoul d become familiar enough with these negative expressi ons that you accur at el y compl et e computations invol ving them In ordinary subtraction, all numbers were positive. Now you will find that subtraction and addition of numbers of either sign frequently occurring when you event ually cal cul ate the days, hours, parts, of the month for a conjunction of the mon.

The next example ill ustrates how this type of problemis worked:


When a number is not preceded by a sign, it is taken to be positive. A few rules apply for these probl ems:
adding two negatives toget her (e.g. -5 +-3) results in a negative number larger than either (-8).
adding a negative number and a positive number together is the same as subtracting. The sign of the l arger val ue (called the absol ute val ue in mathematics) is the sign of the final number.

$$
\begin{aligned}
& (-30+50=+20) \\
& (-48+38=-10)
\end{aligned}
$$

When you subtract a negative number from another negat ive number, you change the sign of the number you are subtracting, and then add as above. $(-5--3=-5++3=-2)$

Adding two negative numbers toget her results in a larger negative number. Subtracting two negative numbers results in a smaller negative number -- closer to zero. Any ti me you are working with negative numbers, be sure that you carry the signs wi th you. Al though you need not specifically mark positive numbers, this may be advisable for you initially in order to keep the si gns clear in your mind. Whenever you are subtracting, remember that the subtraction sign affects every number (term) in the expression:

|  | 45 d | 23 h | 150 |
| :---: | :---: | :---: | :---: |
|  | -8d | 15 h | - 125 p) |

## Kossey

Notice how the numbers subtracted change si gns:

| 45 d | 23 h |
| :--- | :---: |
| +8 d | $-15 \mathrm{~h}+150 \mathrm{p}$ |
| -125 p |  |
| 53 d | 8 h |

Try the following probl em

1. 20
$-18 \mathrm{~d}-2 \mathrm{~h}-780 \mathrm{p}$
$-0 \mathrm{~d}-1 \mathrm{~h}-785 \mathrm{p}$
$-97 \mathrm{~d}-2 \mathrm{~h}-756 \mathrm{p}$
$+74 \mathrm{~d}+14 \mathrm{~h}+196 \mathrm{p}$
$+13 \mathrm{~d}+22 \mathrm{~h}+1284 \mathrm{p}$
$+36 \mathrm{~d}+22$
2. 20 The probl em becomes easier if you total the negative quantities separate from the positive, and then combine.



Thi s reduces (by taki ng one hour and changing it to parts: +1080 - 541) to 8 d 10 h 539 p.

1. 21 Wbrk the next problemwithout reducing until the final step:
$-\binom{210 d-15 h-r i p}{37 d-19 h-1185 p}$

> Change si gns and subt ract.
> Your result shoul d be $173 \mathrm{~d}-34 \mathrm{~h}+214 \mathrm{p}$. This reduces to
> 171 d 14 h 214 p.

1. 22 Add 92 d 581 h 471 p to ei ght times ( $-10 \mathrm{~d}-21 \mathrm{~h}-204 \mathrm{p}$ ). Then subtract $-152 \mathrm{~d}-589 \mathrm{~h} 188 \mathrm{p}$ from the sum Reduce at the final answer. How many days over a full number of weeks is this?

First multiply: - 10 d - 21 h - 204 p x 8

Add:

$$
\begin{array}{r}
-80 \mathrm{~d}-168 \mathrm{~h}-1632 \mathrm{p} \\
92 \mathrm{~d}-581 \mathrm{~h} \\
-171 \mathrm{p} \\
+12 \mathrm{~d}+413 \mathrm{~h}-1161 \mathrm{p}
\end{array}
$$

Subt racting $-152 \mathrm{~d}-589 \mathrm{~h} 188 \mathrm{p}$, you change the signs and add:

$$
\begin{array}{r}
12 \mathrm{~d}+413 \mathrm{~h}-1161 \mathrm{p} \\
+152 \mathrm{~d}+589 \mathrm{~h}-188 \mathrm{p} \\
--164 \mathrm{~d}+1002 \mathrm{~h}-1349 \mathrm{p}
\end{array}
$$

Borrow 2 hours, and then di vi de 1000 hours by 24 to convert the
hours col um to a number less than 24. This gives:
205 d 16 h 811 p
To find how many days this is over a full number of weeks, you di vide by 7 the REDUCED number of days. $205 / 7$ leaves a remai nder of 2 . Two days pl us 16 hours and 811 p is the time over a full number of weeks.

After working the probl ens in this section, you should feel confident about your ability to successfully add, subtract, multiply, and reduce days, hours, and parts. Some may still require additional drill on these oper ations. Desi gn your own problems for more practice. Have another student check your work, or see your instructor for assi stance.

PROGRAM I I

## CALCULATI NG THE DAY OF WEEK FOR MOLAD TI SHRI

## PERFORMANCE GOAL 2:

Given a requi red Roman year, you will correctly cal cul ate the day of the week, hours, and parts for the mol ad Ti shri of that Roman year. Thi s will be accomplished W THOUT CHARTS AND TABLES.

Why do you need to be concerned with the mol ad of Tishri? The answer is that you must know when it occurs bef ore you can det er mi ne the date of the Festival of Trumpets. And all other hol y days withi $n$ a Roman year (J anuary-December) are ultimat y refer enced to that hol y day. The molad of Tishri is prerequisite to most cal cul ations invol ving the Hebrew cal endar. Correct cal cul ation of the mol ad of Tishri is thus essential for determini ng what dates on the Roman cal endar we commonl y use for God's Sacred Festivals to occur.

Certain definitions and concepts need to become crystallized in your mind:

What is a mol ad?
Wen is Ti shri?
What is a bench mark?
How is time reckoned in the cal cul ations?
What is meant by the "advancement " of the mol ad?
Let's find the answers to these questions.
What is the mol ad Tishri?
TI SHRI is the seventh month of the sacred cal endar. The computed time for the conj unction of the sun, moon, and the earth is called a MOLAD, from the Hebrew MDLED ( pl ural, MDLEDOTH). Thi s word means renewal, or rej uvenescence.

Mbl ad of Tishri is the computed time of the new moon of the month of Tishri, whi ch corresponds to Sept ember/October. As Ti shri is al so the first month of the civil Hebrew year, the mol Tishri is al so the cal cul at ed astronomical commencement of the year.

Another term whi ch you will be using is bench mark. All this means is a point of reference from whi ch measurements can be made. Any known mol ad (expressed as day of the month, day of the week, hours, and parts, e. g., Oct ober 6, Sunday, 23 h , 204 p in 3761 BC ) can serve as a bench mark. The most practical choi ce for a bench mark, however, will be the mol ad Tishri of year one in a 19 year cycle. 3761 BC is such a year.

## Kossey

In order to avoid a mental mix-up I ater, l et's clarify how we reckon time. Just what does an expressi on like 4 d 7 h 503 p mean? Such an expressi on can be taken either of two ways: a) as an interval of time b) a time in the week.

Consi der case a). Here, when you are speaking of a length of time, you assume that you are starting froma reference point of 0 days, 0 hours, 0 parts. Thi s is just the same as clocking a track runner on a stop watch that starts ticking at the sound of the gun. A certain time span is i nvol ved, figuring from a bench mark of 0 days, 0 hours, 0 parts.

W th case b), you are still reckoning time, but froma DI FFERENT reference point. In the cal endar cal cul ations you are working in Theol ogi cal Research, the week begi ns at Sunday midni ght. Sunday is regarded as the first day of the week; midni ght the zeroth hour, zero parts:

The week begins: Sunday: 1 d 0 h 0 p .
An expressi on like 1 day 13 hours 0 parts MUST BY DEFI NI TI ON OF THE STARTI NG POI NT refer to Sunday, the 13th hour after midni ght, or Sunday 1 PM

Li kewi se, 1 day 13 hours 179 parts refers to a time slightly later than 1 PM on Sunday.

The reason why we arbitrarily begi $n$ Sunday as 1 day 0 hours 0 parts is so that there will be an exact coinci dence with the days of the week: 1 d is Sunday; 2 d is Mbnday; 4 d is Wednesday; 7 d is Sabbath. So long as all the numbers are reduced, you merely look at the number of days, and these correspond to the day of the week. (For the same reason of SI MPLI CITY, we begi $n$ the day with midni ght instead of 6 PM so far as the CALCULATI ONS are concerned. Si nce the bench mark is expressed by this same reckoning, the final results will be exactly the same.)

By contrast, if we deci ded to define the begi nni ng of the week, Sunday, as the zero day, and Sat urday sunset (say 6 PM ) as the start of Sunday, we woul d have the expressi on 0 d $0 \mathrm{~h} 0 \mathrm{p}=6$ PM Sat ur day eveni ng. Then 1 d 13 h 179 p would have an entirely different meaning, if it referred to a time in the week. 1 d would then be Monday; 13 h 179 p would be slightly after 7 AM Do you see how mach more compl i cated your work woul $d$ then become?

Here is another di stinction that can hel p you understand the difference in meaning bet ween an interval of time, and a time in the week:
a) an interval of time:
parts are added to the hours
are added to the days
b) a time in the week:
parts are added to the hours
W THI N the day of the week.
If 4 days 7 hours 503 parts refers to an interval of time, then 503 parts pl us 7 hours is added to 4 days. Ther ef ore the interval of time goes as far as the 7 th hour(and 503 parts) of the fifth day.

If 4 days 7 hours 503 parts is to be taken as a point in the week, then 503 parts pl us 7 hours is WTHN the 4 th day of the week.

Remember this difference in what is meant comes about by the way the starting point is DEFI NED, and for no other reason.

During your cal cul ation of the advancement of the mol ad over a

## Kossey

full number of weeks, you may come up with a number like 0 days 4 hours and 71 parts. Be conf orted by the fact that the expressi on is si mply an i nterval of time by the way Sunday midni ght is defined.

How can you rel ate a time interval to an expression of time during a week?

YOU MUST ADD AN INTERVAL OF TI ME to the BENCH MARK bef ore you can determine a real day of the week. The bench mark will impicitly tell you where the week begi ns. The bench mark for the year 3761 BC is Sunday, the 23rd hour 204 parts. (We could have called that bench mark Mbnday if we started the day at 6PM instead of midni ght. But then we woul d be confronted with a more compl icated interpretation of what the time expressi ons mean.)

Now transformeach of the following expressi ons to a) a time interval b) a time during the week. Use thi s format:
a: $\qquad$ days, pl us $\qquad$ hours $\qquad$ parts of the $\qquad$ day
b: (day of the week), bet ween $\qquad$ \& $\qquad$ AM or PM

| 2. 1 | 4 | $d$ | 3 | $h$ | 191 |
| :--- | :--- | :--- | ---: | ---: | ---: |
| 2. 2 | 1 | $d$ | 16 | $h$ | 304 |
| 2. 3 | 6 | $d$ | 7 | $h$ | 8 |
| 2. | $p$ |  |  |  |  |
| 2. 4 | 7 | $d$ | 22 | $h$ | 5 |
| 2. 5 | 2 | $d$ | 13 | $h$ | 871 |

The answers are gi ven bel ow.
2. 1 a) 4 days pl us 3 hours 191 parts of the 5 th day
b) Wednesday, bet ween $3 \& 4 \mathrm{AM}$
2. 2 a) 1 day pl us 16 hours 304 parts of the 2nd day
b) Sunday, bet ween $4 \& 5 \mathrm{PM}$
2. 3 a) 6 days pl us 7 hours 8 parts of the 7 th day
b) Friday, bet ween $7 \& 8$ AM
2.4 a) 7 days pl us 22 hours 5 parts of the 8 th day
b) Sabbath, bet ween 10 \& 11 PM
2.5 a) 2 days pl us 13 hours 871 parts of the $3 r d$ day
b) Monday, bet ween 1 \& 2 PM

One particularly important time int erval will occur throughout your experi ence with the Hebrew cal endar. In order to cal cul ate the mol ad Tishri, you will be working with two molads:

* a known mol ad, such as 3761 BC -- a bench mark
* the mol ad of the Roman year whi ch you are det er mining

The time interval bet ween these two mol ads is the ELAPSED TI ME. Si nce you are deal ing with the mol ad of Tishri in both the required year and the bench mark, the el apsed time will be a whole number of years, such as 4520 years, 1503 years, 38 years.

Let's i nvestigate anot her feat ure of the Hebrew cal endar whi ch we can cal I MOLAD "ADVANCEMENT." Now you know that the MDLAD itsel f doesn't really move, since the termis defined as the cal cul at ed conj unction of the sun, mon, and the earth. We are using a figure of speech very simiar to sun "rise."

Here's an ill ustration of what is meant by the advancement of the mol ad. In 3761 BC , the mol ad Ti shri was on Sunday 1 d 23 h 204 p . In 1980 AD the mol ad Ti shri will be Tuesday 3 d 23 h 206 p . Al though thousands of years have el apsed over the time span, the APPARENT "advancement" in the week of the second mol ad is only 2 days 0 hours and 2 parts.

Kossey
Why does the mol ad occur on different days of the week? The length of an aver age I unar month is 29 d 12 h 793 p . How much great er than four full weeks is this?


The mol ads of two successi ve mont hs cannot occur on the same day of the week because of this EXCESS over 28 days. In one month, if the mol ad were Mbnday 8 AM ( 2 d 8 h 0 p ), the mol ad of the next month woul d be:

```
    2 d 8 h 0p
    +(1 d 12 h 793 p)
    3 d 20 h 793 p, or Tuesday, bet ween }8\mathrm{ and 9 PM
```

Al though the second mol ad occurred 29 d 12 h 793 p after the first one, the second mol ad was "di spl aced" WTH REFERENCE TO THE WEEK by 1 d 12 h 793 p . Merely as a conveni ent label, we will ref er to that apparent shift as MDLAD ADVANCEMENT. But the mol ad doesn't move; onl y the time of its occurrence IN THE WEEK apparently advances.

The TOTAL MOLAD ADVANCEMENT i s si mpl y the EXCESS over the number of full weeks in the el apsed time from the bench mark to the mol ad Tishri of the desired Roman year.

Just as monthly mol ads will occur on different days, the mol ad of Ti shri will advance in the week over the previ ous mol ad of Ti shri. If in three years the tot al advance were 13 days, the Mblad woul d be 13 days later. But in terms of the day of the week, $t$ his would be 13-7, or 6 days later in the week.

Now you will learn how to cal cul ate the day of the week for the mol ad Ti shri.

You will find it easier to understand how to determine the day of the week for a gi ven mol ad if the steps are explai ned first without the mathematical details:

In order to determine the day of the week for the mol ad of Tishri, you must find the TOTAL ADVANCEMENT of the mol ad that occurs within the time span i nvol ved from a known mol ad, or bench mark.

What will make up that time span? From that bench mark, a certain number of years will el apse to the particular year in question:
mol ad of Tishri mol ad of Tishri (bench mark) of requir red year
(el apsed time)

Later on in this program you will I earn how to express el apsed time as: a whole multiple of 19 year cycl es,
pl us the excess number of common years,
pl us the excess number of leap years.
Logi cally, the tot al advancement of the molad, or excess over full weeks in the el apsed time, for the year in question can be found by addi ng:
the advancement due to whol e multipl es of 19 year cycles, + the advancement due to the number of common years,

```
Kossey
+ the advancement due to the number of leap years.
```

An example will clarify this. If the el apsed time is 156 years, there are 8, 19 year cycles, 4 common years, and 2 leap years (you'll see how this is done later). The total advancement of the mol will be:

$$
\begin{aligned}
& 8 \text { times the advancement of one } 19 \text { year cycl e } \\
&+ 4 \text { times the advancement of one commn year, } \\
&+2 \text { times the advancement of one leap year. }
\end{aligned}
$$

The final step is si mply adding the tot al excess over full weeks to what ever bench mark you started with. This figure will give you the day of the week of the mol ad Ti shri in question.

Be sure that you understand the qual itative el ements connected with the advancement of the molad. A mol ad "advances" with respect to a known mol ad because of the excess time in one aver age I unar month over a full number of weeks. Al I you are doing is adding the tot al advancement that occurs within the time interval from the bench mark to the mol ad Tishri of the year in question. You are actually finding the excess over the full weeks from the bench mark to the mol ad of the requi red year.

Your success at determing the day of the week of a requi red mol ad i mpinges upon:
a) Correctly finding the ELAPSED TI ME froma bench mark to the requi red year.
b) Expressing the el apsed time in terms of multiples of 19 year cycl es

$$
\begin{aligned}
& +\quad \text { common years i n the remai nder } \\
& +\quad \text { leap years in the remai nder }
\end{aligned}
$$

c) Cal cul ating the mol ad "advancement" attributable to each el ement of $b$ ), then adding these toget her, reducing as necessary.
d) Adding the reduced advance of the mol ad c) to the bench mark.

## ELAPSED TI ME, 2A

Three possibilities exist for the el apsed time from the bench mark to the required year:
a1) Both years are AD dates.
a2) Both years are BC dates.
a3) One year is $B C$ and the ot her is $A D$
(Al though you might work backwards in time, this program will only consi der problems in which the bench mark is the EARLIER of the two years.)
a1: Bench mark and required year both $A D$ dat es. The el apsed time is si mply the difference bet ween the two years. If the bench mark is 1845 AD and the requi red year is 1931, the el apsed time is:

$$
1931-1845=86 \text { years }
$$

If the bench mark is 895 AD and the requi red year is 1751, the el apsed time is:

$$
1751-895=856 \text { years }
$$

a2: Both years are BC dates. Many cal cul ations use 3761 BC as a bench mark. To be mathematically consi stent, it is hel pf ul to pl ace a negative si gn ( - ) bef ore all BC years. Si nce you will still be


#### Abstract

Kossey subtracting in order to find the difference, the second number will become positive - ( ) $=+$. As you are primarily concerned with years after 3761 BC , or -3761, the year in question will al ways be more positive (closer to zero). As a check when both years are BC, expect the el apsed time to be SMALLER than 3761 years. (Use onl y the "absol ute val ue" for the el apsed time, whi ch means you can di sregard the final negative si gn.)


Using 3761 BC as a bench mark, what is the el apsed time to 585 BC ?

$$
-3761-(-585)=-3761+585=-3176
$$

The el apsed time is 3176 years.
What is the el apsed time to 1486 BC?

$$
-3761-(-1486)=-3761+1486=-2275
$$

The el apsed time is 2275 years.
a3: Bench mark is BC and the requi red year is AD. 3761 BC is
frequently used as bench mark for AD years. Onl y one thing is really different here. There is no year allot ed for 0 AD or 0 BC.
Mat hematically, the number system does have a 0 . What do you do?
Graphically, the two systems look like:
$\begin{array}{lrlrlllllllllllll}\text { Cal endar : } & 5 & B C & 4 & B C & 3 & B C & 2 & B C & 1 & B C & 1 & A D & 2 & A D & 3 & A D\end{array}$
The number scale has one more place, a zero, than the cal endar.
Ther ef ore in time intervals that cross AD/ BC, you must SUBTRACT ONE from the answer you compute arithmetically. (The time span from 4 BC to 2 AD i s onl y five years.) BE SURE YOU REMEMBER TO SUBTRACT ONE FROM THE MATHEMATI CAL COMPUTATI ON! But onl y when you cross over from BC to AD.

What is the time el apsed from 3761 BC to 1000 AD?

$$
-3761-+1000=-3761-1000=-4761
$$

The el apsed tire, however, is 4761 - 1 year, or 4760 years.
What is the time span (el apsed time) to 1974 AD from 3761 BC?

$$
-3761 B C-(+1974)=-3761-1974=-5735
$$

The el apsed time is 5735 - 1 , or 5734 years. As a check, you shoul d be aware that in going from $B C$ to $A D$, the el apsed time will be a larger number ( $t$ aken as an absol ute val ue) than either the bench mark or the requi red year.
Drill yourself on el apsed time calcul ations with the following
probl ens. Remenber that if you are wrong here, all the rest of your cal cul ations will be off!

Bench mark Requi red year El apsed time

| 2a. 1 | 1845 AD | 1971 AD |
| :--- | ---: | ---: | ---: |
| 2a. 2 | 1883 AD | 1945 AD |
| 2a. 3 | 3761 BC | 1442 BC |
| 2a. 4 | 3761 BC | 4 BC |
| 2a. 5 | 3761 BC | 721 BC |
| 2a. 6 | 1861 BC | 604 BC |
| 2a. 7 | 3761 BC | 31 AD |
| 2a. 8 | 3761 BC | 1859 AD |
| 2a. 9 | 3761 BC | 1982 AD |

See problems 2 b. 1 to 2 b .9 for the el apsed time.

Once you' ve det ermined the number of years bet ween the bench mark and the requi red year, you need to express that time in terms of the Hebrew cal endar.

On a practical basis, 19 year cycles are conveni ent. Find the number of 19 year cycl es in the el apsed time, divide the el apsed time by 19. The QUOTI ENT is the number of 19 year cycles.

Usually, however, this division will result in a remai nder, a number from' 1 to 18 . This remai nder will tell you the number of years el apsed in the next cycle.

As you al ready know each 19 year cycle consists of 12 common years and 7 I eap years.

| SI NCE | 142 | AD (see foot note 1$),$ | the years in a cycle that are leap |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| years are: | 3 | 6 | 8 | 11 | 14 | 17 | 19 |  |

Be sure to nemorize these numbers! (Of course, the years 1, 2, 4, 5, 7, $9,10,12,13,15,16$, and 18 are common.)

Bef or e 142 AD , the I eap years were: $\quad 2 \begin{array}{lllllll}5 & 7 & 10 & 13 & 16 & 18\end{array}$

For any remai nder you acqui re after di viding the el apsed time by 19, you:

1) Deci de whet her the requi red year is bef ore 142 AD , or 142 AD and after.
2) Count the number of leap years that fit in the remai nder (or you could count the common years).
3) The number of common years will be the remai nder mi nus the number of leap years.

For exampl e, the el apsed time from 1845 AD to 1975 AD is 130 years. This is $130 / 19$ time cycles, or six 19 year time cycles pl us 16 years remai nder.

The I eap years for 1975 (after 142 AD) are 3, 6, 8, 11, 14. Fi ve leap years al together. Si nce 1975 has 16 el apsed years and there are five leap years thus far, there must be 16 - 5 or 11 common years.
(foot note 1) There i s some evi dence that an adj ust ment to the Hebrew cal endar may have taken place during the patriarchate of Si mon III (140-163). See Cyr us Adl er, "Cal endar, Hi story of," in "The J ewi sh Encycl opedi a" (New York: Funk and Kagnalls, 1907), Vol. 3, p. 500.

Here is another example. If the bench mark is 3761 BC and the required year is 27 AD, the el apsed time is:
$-3761-(+27)=-3788$ years; the el apsed time is 3787 years, si nce you go from $B C$ to $A D$.

3787 / $19=19919$ year cycles, plus 6 el apsed years.
For 27 AD (bef ore 142 AD), the leap years of the cycle are 2 and 5 . With two l eap years, there must be $6-2$, or 4 common years.

Practice expressing el apsed time in terms of 19 year cycles, the number of leap years, and the number of common years. See problems $2 a .1$ to 2a. 9.

| El apsed | Kossey |  |
| :---: | :---: | :---: |
| time | 19 year <br> cycl es | \# of l eap |$\quad$ \#ears common


| 2b. 1 | 126 yrs |
| :--- | ---: |
| 2b. 2 | 62 |
| 2b. 3 | 2319 |
| 2b. 4 | 3757 |
| 2b. 5 | 3040 |
| 2b. 6 | 1257 |
| 2b. 7 | 3791 |
| 2b. 8 | 5619 |
| 2b. 9 | 5742 |

Check answers agai nst those bel ow.

| 2b. 1 | 6 | cycl es | 4 |
| :--- | ---: | :--- | :--- |
| 2b. 2 | 3 | leap | 8 common |
| 2b. 3 | 122 | 1 | 4 |
| 2b. 4 | 197 | 0 | 4 |
| 2b. 5 | 160 | 5 | 9 |
| 2b. 6 | 66 | 0 | 0 |
| 2b. 7 | 199 | 1 | 2 |
| 2b. 8 | 295 | 4 | 6 |
| 2b. 9 | 302 | 5 | 9 |

"ADVANCEMENT" OF THE MOLAD. 2c
From your previ ous program (1A \& 1B), you are equi pped to find out the requi red inf ormati on regarding the advancement of the mol ad. Si nce a I unar month has 29 days, 12 hours 793 parts, it is 1 day, 12 hours, and 793 parts in excess of a full number of weeks. As you saw bef ore, a mont hly mol ad (two successi ve mont hs) advances 1 d 12 h 793 p.

Knowing this, you can easily det ermine how much a mol ad "advances" in a common year of 12 mont hs, in a leap year of 13 mont hs, or in a 19 year cycle of 235 months.

How much does the time of a mol ad "advance" in the week during a common year?


After 12 months, the mol ad "advancement" is 18 d 8 h 876 p. Si nce full weeks will not affect the days of the week, you can di vi de the reduced number of days by 7. So with respect to the week the mol ad "advancement" for a commo year is:

$$
4 \mathrm{~d} 8 \mathrm{~h} 876 \mathrm{p} .
$$

You should notice an alternative way of arriving at the same number. How much does a common year exceed the number of full weeks in the year? Multiply the length of an average I unar month by 12 months:

$$
12 \times(29 \mathrm{~d} 12 \mathrm{~h} 793 \mathrm{p})=348 \mathrm{~d} 144 \mathrm{~h} 9516 \mathrm{p}
$$

Reduce this number: 354 d 8 h 876 p

Kossey
Di vi de the reduced number of days by 7 to eliminate full weeks:

$$
4 \mathrm{~d} 8 \mathrm{~h} 876 \mathrm{p} .
$$

2c. 1 Verify that an average leap year exceeds a full number of weeks by 5 d 21 h 589 p .
2c. 2 Verify that a 19 year cycle exceeds a full number of weeks by 2 d 16 h 595 p .

From here on, you al ready are competent to handle the det ails of multipl ying the advancement of the mol ad, and then adding. For 4 BC (as in 2a. 4 and 2 b. 4) you will do the following:


Multiplying and adding you will di scover that this is:
567 d 5228 h 254, 404 p.
When reduced, and di vi ded by seven (onl y the full reduced days are di vi ded by 7), the advancement of the mol ad over a week is:

$$
3 \mathrm{~d} \quad 15 \mathrm{~h} \quad 604 \mathrm{p} .
$$

## ADDI NG REDUCED ADVANCEMENT OF MOLAD TO THE BENCH MARK, 2d

The bench mark for 3761 BC is Sunday 23 h 204 p . This is reckoni ng from midni ght. As Sunday is the first day of the week, the bench mark can be expressed as:

1 d 23 h 204 p.
From here on, you merel $y$ add the reduced advancement of the mol ad to the bench mark. For 4 BC , this is:


The fifth day of the week is Thursday, so the mol ad occurred on Thursday, the 14 th hour ( 2 PM ), 808 parts.

So I ong as the final d, h, p, are reduced, you need onl y be concerned (for now) with the first colum.

2d. 1 Verify that the mol ad or 721 BC is 5 d 7 h 364 p.
2d. 2 Verify that the mol ad or 31 AD is 5 d 23 h 941 p.
2d. 3 At this point, you should test yourself to be sure you can fulfill the goal of program 2. Cal culate the DAY OF THE WEEK of the mol ad Tishri for 1996 AD wi thout consulting this program You'll need to have memorized (or el se work out agai $n$ ) the mol ad "advancement" for a 19 year cycle, a common year, and a leap year, as well as remember a bench mark.

Compare your cal cul ations to the answer bel ow, which is worked out in det ai I.

Cal cul ate the DAY OF THE WEEK of the mol ad Ti shri for 1996 AD.




| 128 weeks |  |  | 218 d |  | 179 h |  |  | (reduci ng) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $7 /$ | $\begin{aligned} & -\quad-\quad \\ & 900 \\ & x \times x \end{aligned}$ | 24 / | $\begin{aligned} & 5233 \\ & x \times x \end{aligned}$ |  | 1080 | $\underset{\text { xxx }}{193736}$ | p |  |
|  | 4 d |  | 1 |  |  | 416 | p | reduced advancement of molad |
|  | + (1 d |  | 23 |  |  | 204 | p | bench mark |
|  | 5 6 6 |  | 24 |  |  | 620 | p |  |

The day of the week of mol ad Tishri in 1996 AD is Friday.

## PERFORMANCE GOAL 3

W thout tabl es, you will correctly determine the day of the month of the mol ad Ti shri for the Roman years specified in this program

The procedure used to find the day of the month of the mol ad Ti shri parallels that of the day of the week cal culation you al ready

## Kossey

I earned in Program 2. Part of the work needed for the day of the mont h cal cul ation is ALREADY accompl ished in the day of the week cal cul at ion!

Let's revi ew part of the day of the week cal cul ation to see just what operations are in common. You must have a requi red year and the bench mark (assumed to be 3761 BC unl ess ot her wi se specified). From these two years, you can det ermine el apsed time bet ween them (read over 2a again if this is hazy). Be sure you remember to subtract 1 fromthe total whenever you go from BC to AD. This is a common mistake!

Next, you express the el apsed time in terms of 19 year cycles, number leap years, and the number of common years (2b):

* $\quad$ Di vi de the el apsed ti me by 19.
* The remai nder is the number of el apsed years in the 19 year cycle of the required year. If the requi red year is 142 AD or after, the intercal ary years are $3,6,8,11,14,17$ and 19. (Bef ore 142 AD, just subtract 1 fromeach of these numbers to obtai $n$ the intercal ary years of that cycle.)
* Count up the number of leap years that have occurred within the cycle, including the l ast number. For example, if the remai nder is 11, there are 4 leap years in the cycle.
* Subtract the number of leap years fromthe remai nder, and you will have the number of common years in that cycle.

You have al ready done this much of the cal cul ation in order to det er mine the day of the week for the mol ad of Tishri. And you will use thi s SAME information to find the day of the month. How is it applied?

The answer rests in the basic difference bet ween the Hebrew year and the Roman (Julian) year. Common years in the Hebrew cal endar have 353,354 , or 355 days, wher eas a Roman year has $3651 / 4$ days. Compar ed to the Juli an year, the Hebrew common year falls short. A Hebrew i nt ercal ary year ( 383,384 , or 385 days) is longer than the Roman year.

Exactly how much will the Hebrew cal endar trail the Roman cal endar in a common year of 12 mont hs? The aver age commn year is 12 times ( 29 $\mathrm{d} 12 \mathrm{~h} 793 \mathrm{p})$. This is 354 days, 8 hours and 876 parts.

$$
\begin{aligned}
& 365 \mathrm{~d} \quad 6 \mathrm{~h} \quad 0 \text { parts (average Julian year) } \\
& \text { - ( } 354 \mathrm{~d} 8 \mathrm{~h} 876 \mathrm{p} \quad \text { (aver age common year) } \\
& 10 \text { d } 21 \text { h } 204 \text { p }
\end{aligned}
$$

In ot her words, the average common year of the Hebrew cal endar is 10 d 21 h 204 p LESS than an aver age $J$ ul i an year. For cal cul at i on pur poses, remenber the number as $-10 \mathrm{~d}-21 \mathrm{~h}-204 \mathrm{p}$. Thi s is the same as $-(10$ d 21 h 204 p ). If you cannot recall this number under the pressures of a test, you should remember HOW it is found.

How much LONGER is an aver age int er cal ary (or leap) year than the aver age Jul i an year? To find the length of an aver age int er cal ary year, multiply 13 months times ( 29 d 12 h 793 p ).

$$
\begin{aligned}
& \text { - (365 d } 6 \mathrm{~h} \quad 0 \mathrm{p} \text { ) (average Jul i an year) } \\
& 18 \text { d } 15 \text { h } 589 \text { p }
\end{aligned}
$$

In an intercal ary year, the Hebrew cal endar EXCEEDS the Jul i an cal endar by $+18 \mathrm{~d}+15 \mathrm{~h}+589$. The pl us signs are carried in order to mi ni me conf usi on. As you can see, this number is very easy to find.

Kossey
What about the 19 year cycle and the two cal endars?


How much shorter is the 19 year cycle of 235 I unar months than 19 Julian years?


Express this as $-1 \mathrm{~h}-485$ p. A 19 year cycle is one hour and 485 parts LESS than 19 Juli an years.

Let's summarize these three i mortant numbers. Be sure you understand exactly what each means!

$$
\begin{array}{r}
0 d-1 h-485 p \\
-10 d-21 h-204 p \\
+18 d+15 h+589 p
\end{array}
$$

19 year cycle is SHORTER than 19 Jul i an years.
average common year is SHORTER than an average Julian year.
average leap year is LONGER than an average Julian year.

You use these three numbers quite like the three ot her numbers you worked with in cal cul ating the day of the week for the mol ad Tishri. The onl y complication is that some of these numbers are negative, and you must be certain that you do not overlook a negative number by assuming it is positive. It's safer and surer to carry the signs al ong through every step.

Now let's ill ustrate a cal culation of the day of the month for the mol ad Ti shri. Be aware of the fact that the years called for in this program avoid certain complications, which will be explai ned in program 5.

What is the day of the month of the mol ad Ti shri in 1520 AD? Proceed as you would for finding the day of the week.


For the day of the month, you are determing how far the Hebrew cal endar lags or l eads the Roman cal endar:

$$
\begin{gathered}
-1 \mathrm{~h} \quad 485 \mathrm{p} \quad(1 \mathrm{ag} \text { per } 19 \mathrm{yr} \mathrm{cycle}) \quad-10 \mathrm{~d} \quad-21 \mathrm{~h} \quad-204 \mathrm{p} \\
\text { Page } 26
\end{gathered}
$$



The amount the Hebrew cal endar will LAG behi nd the Roman during the el apsed time is the sum of the amounts trailed in the common years and the 19 -year cycl es:

| - 110 d | $\begin{aligned} & -231 h \\ & -277 h \end{aligned}$ | $\begin{aligned} & 2244 p \\ & -134345 p \end{aligned}$ |  |
| :---: | :---: | :---: | :---: |
| -110 d | - 508 h | - 136589 p | Ant. Hebrew cal endar LAGS Roman cal endar during el apsed time. |
| +108 d | $+90 \mathrm{~h}$ | + 3534 p | Ant. Hebrew cal endar I eads Roman cal endar during el apsed time. |

$$
-2 \mathrm{~d} \quad-418 \mathrm{~h} \quad-133055 \mathrm{p}
$$


$-24 \mathrm{~d}-13 \mathrm{~h}-215 \mathrm{p}$ represents the TOTAL AMOUNT that the Hebrew cal endar trails the Roman cal endar. (Just reduce the number to a conveni ent negative form full reduction isn't necessary.) NEVER di vi de this number by 7! To find the day of the month, add the lag you have cal cul at ed to the bench mark. Si nce Septentor has 30 days, you can express Oct. 6 as Sept. 36.

Sept entorer $\begin{array}{r}36 \\ +(24 d-13 \\ 24 \\ \text { h }-215 \\ \text { p })\end{array}$


The mol ad Tishri in 1520 AD was on September 12, just bef ore 10 AM
The procedure for finding the day of the month for the mol ad Ti shri is al most exactly parallel to that for finding the day of the week. To gi ve you an overvi ew of both cal cul at i ons, here in schematic form are the steps. Read this chart fromleft to right, as well as down the page.

| Mbl ad Ti sh | Day of the week | Day of the month |
| :---: | :---: | :---: |
| Bench mark \& requi red | 3761 BC - requi red yr | 3761 BC - requi red yr |
| year. Going from BC | (-1)? | (-1?) |
| to AD? El apsed time |  |  |
| in years: (di vi de by 19) |  |  |
| number of 19 yr cycl es | times (2d 16h 595p) | times ( $-1 \mathrm{~h}-485 \mathrm{p}$ ) |
| number of common years | times (4d 8h 876p) | " (-10d - 21h - 204p) |


| number of I eap years | $\begin{array}{r} \text { Kossey } \\ \text { times (5d } 21 \mathrm{~h} 589 \mathrm{p}) \end{array}$ | " ( +18d +15h +589p) |
| :---: | :---: | :---: |
|  | Sumis the mol ad "advancement," or excess over full weeks, in the el apsed time. ( Di vi de the REDUCED days by 7.) | Sumis the amount that the Hebrew cal endar lags (-) or l eads ( +) the Roman cal endar. (Reduce as necessary.) |
| Add to the bench mark | +(1 d 23 h 204 p) Oct. | $6 \text { ( Sept 36) 23h 204p }$ <br> ections, in program 5] |
| Answer : | (* day of week) | (* day of month) |

In most cases, you will determine the day of the week first, as there is less chance for computational errors. This cal cul ation will gi ve you a certain number of hours and parts besi des the day of the week.

Is there any way of knowing that your cal cul ations are correct? Yes! The hours and parts for the day of the week of the mol ad Tishri must be i dentical to the hours and parts for the day of the month. For 1520 AD, the example above, the hours and parts for the day of the week are al so 9h 1069p. Remember this rule for checking your work.

Si nce the format invol ved in cal cul ating the day of the month is so very similar to the day of the week, a large amount of additional practice will not be necessary. You should, however, work out a few probl ens.
3. 1 What is the day of the month of the mol ad Tishri for 1492 AD?
3. 2 What is the day of the month of the mol ad Tishri for 28 AD?

The problens are partially worked out for you bel ow.
3. 11492 AD is 5252 el apsed years. This is 27619 year cycles, and 8 el apsed years. Ther ef ore there are 3 l eap years $(3,6,8)$ and 5 common years. The mol ad was on September 21, 19 hours and 1011 parts. (Thi s was a Friday, in case you want to check that cal culation, too.)
3. 228 AD is 3788 el apsed years. 3788 years is 19919 year cycl es and 7 years. In 7 el apsed years, there are 3 leap years ( $2,5,7$ ) and four common years. (If you noticed that these multipliers for the leap and common years are the same as in 3.1, you saved yourself some work!) The mol ad was on Oct ober 7, 8 hours 760 parts. (Thi s was a Thursday.)

Now that you have al most compl et ed this program you shoul d feel confident of your ability to cal cul ate the day of the month for the mol ad Tishri. If you experi enced a bit of difficulty with problems 3.1 and 3. 2, you may want to work out 1520 AD agai $n$, and then compare each of your steps with the example al ready worked above.
3. 3 As a final self-test, without consulting the program or your notes, determi ne the day of the month of the mol ad Ti shri for 1448 AD. The answer is worked out in detail bel ow.

Cal cul ate the day of the month of the mol ad Tishri for 1448 AD:


Page 28


| Sept 0 | $=$ August 31 |  |
| :--- | :--- | :--- |
| Sept -1 | $=$ | August 30 |
| Sept -2 | $=$ | AUGUST 29 | day of the mont $h$

PROGRAM I V
USI NG TABLES TO FI ND THE MOLAD TI SHRI

## PROGRAM GOAL 4

Given a table of reduced days; hours, and parts, for the Hebrew cal endar, you will correctly calcul ate the day of the week and the day of the Roman month for the mol ad Tishri of sel ected years. Use of the table will consi derably shorten the time needed.

Having worked out the problens in the first three prograns of this series, you may have deci ded that Hebrew cal endar cal cul ations are more tedi ous than difficult.

Yes, some of the arithmetical operations tend to be time consuming. Per haps you feel that you can cal cul at e the mol ad Ti shri quite well, but you'd prefer havi ng some ki nd of a desk top computer just to save time and frustration! Not that all the steps are too i nvol ved for you to do yourself; only the lengthy multiplication and di vi si on.

Finding the el apsed time is just a qui ck subtraction or addition operation. Expressing the el apsed time in terms of 19 year cycles still isn't demanding. The real hang-up comes in the multiplication of the 19 year cycles, the leap years, and the common years, right?

Mbst of that step, so far as the multiplication is concerned, can be el im nated by using a table of reduced days, hours, and parts. You will be able to calculate the mol Tishri in less than half the time
previ ously requi red!
Exam ne part I of the chart incl uded in this program On the left side of the page as you read it is a table marked " 19 YEAR TI ME CYCLES. " Toward the bot tom of the page are two ot her tables "I NTERCALARY YEARS" and "COMDN YEARS." (We'll ret urn to the ot her table on part l a little later.)

As you suspect, each of these three tables will replace the multiplication operations you did bef ore in order to find the excess over a number of full weeks, or the mol ad "advancement." Now you can read a reduced number from the table very conveni ently

Suppose you have a probl emin which the el apsed time is 20019 year cycles, three leap (intercal ary) years, and five common years. What will be the excess over full weeks?


Of course, most of your cal cul ations of el apsed time will invol ve inter medi ate val ues of 19 year cycles, which are not directly on the first table. What do you do with 189 cycles, 315 cycles, and the like? Just add them with the units you al ready have. If, in the example above, we had 27619 year time cycl es instead of 200, you would add the mol ad "advancement" for 70 cycl es and for 6 cycl es:


You can add these numbers in less time than it takes to multiply $276 \times(2 \mathrm{~d} 16 \mathrm{~h} 595 \mathrm{p})$. Notice that you can performall your addition in one step.

For the 19 year cycles, it may be easi er in some problems to do a subtraction of $t$ wo numbers in the tablerather than an addition of three. The excess over full weeks of 29719 year cycl es can be found either way:

\begin{tabular}{|c|c|c|c|c|}
\hline $$
\begin{array}{lr}
\text { add } & 200: \\
& 90 \\
7:
\end{array}
$$ \& $\begin{array}{ll}5 & d \\ 4 \\ 4 & d \\ 4 & d\end{array}$ \& 22
1
19

19 \& $$
\begin{aligned}
& 200 \mathrm{p} \\
& 630 \mathrm{p} \\
& 925 \mathrm{p}
\end{aligned}
$$ \& <br>

\hline 297 cycl es: reduced: \& 13 d

0 \& $$
\begin{aligned}
& 42 \mathrm{~h} \\
& 19 \mathrm{~h}
\end{aligned}
$$ \& \[

$$
\begin{array}{r}
1755 \mathrm{p} \\
675 \mathrm{p}
\end{array}
$$
\] \& <br>

\hline subtract $\begin{gathered}300 \text { : } \\ -3:\end{gathered}$ \& \[
$$
\begin{array}{rr}
1 & d \\
-1 & d
\end{array}
$$

\] \& \[

$$
\begin{gathered}
21 h \\
-1
\end{gathered}
$$

\] \& \[

$$
\begin{array}{r}
300 \mathrm{p} \\
-705 \mathrm{p}
\end{array}
$$

\] \& \[

p\left($$
\begin{array}{lllll}
p & d & 20 & h & 1380
\end{array}
$$\right)
\] <br>

\hline \& 0 d \& 19 \& 655 p \& <br>
\hline
\end{tabular}

Now practice using the table to find the day of the week of the mol ad Tishri for:
4. $1 \quad 1520$ AD. The probl em is worked out bel ow.


Using the table:


Mbl ad Tishri: Wednesday 1520 AD
The first time through a calculation with the chart took more time than you will need later. But we can even go through a problemfaster by using the table on the first page called "ELAPSED YEARS IN ONE 19 YEAR CYCLE."

Thi s table conbi nes the leap years and the common years of a 19 year cycle toget her, both for dates bef ore 142 AD and for those after. Once you di vi de the el apsed time by 19, you can mat ch the remai nder di rectly wi thout figuring leap years and commn years separately. If your remai nder is 12 (for a year after 142 AD), the excess over ful weeks woul d be 2 d 12 h 724 p .
4. 2 Re-cal cul ate the day of the week for the mol ad Tishri for 1520 AD using the combi ned table. The answer is bel ow.

```
3 7 6 1 ~ B C
1520 AD
5281; 5280 years
```



From the chart, part I:

| 200 | cycl es: | 5 d | 22 h | 200 p |
| :---: | :---: | :---: | :---: | :---: |
| 70 | " | 6 d | 6 h | 610 p |
| 7 | " | 4 d | 19 h | 925 p |
| 17 | el apsed years: | 6 d | 10 h | 210 p |
| Tot a |  | 21 d | 57 h | 1945 p |
| benc | h mark | 1 d | 23 h | 204 p |
|  |  | 22 d | 80 h | 2149 p |
| Redu | ced: | 4 d |  | 1069 p |

Mbl ad Ti shri: Wednesday.
Notice that you need not reduce until the last step. If you want, it's even possible to add the benchmark after the el apsed years of a

## Kossey

cycle-without sub-totaling first. Once you thoroughl y understand the concepts i nvol ved in cal cul ating the mol ad Tishri, you can make these economi es in your work.
(Inci dent ally, if you choose to add the bench mark al ong with the numbers fromthe table, your answer -- after reduction - may take the form of $0 d x \times h \quad x x x$. What day of the week is 0 d ? Just add 7 to 0 d, and you will have the real day - the sabbath. When you are working with the days of the week, "7" is the "additive complement" of numbers from-6 to 0. There's not hing mysterious here, just a mathematical "I aw".)

Now look at part II of the Hebrew cal endar chart. Thi s section will assist you in cal culating the day of the month. The tables are very much like those of part 1. The negative numbers represent the amount the Hebrew cal endar lags behi nd the Roman cal endar for a gi ven unit of time. The positive numbers indi cate that the Hebrew cal endar I eads the Roman.

Because of the different signs invol ved, you'll find it easi er to t ot al the negati ve numbers toget her bef ore combining the positive. For example, how much does the Hebrew cal endar trail the Roman after 276 19 year cycles pl us 8 el apsed years?

| 200 cycl es: | -12 d | 1 h | - 880 p |
| :---: | :---: | :---: | :---: |
| 70 cycl es: | - 4 d | - 5 h | - 470 p |
| $6 \mathrm{cycl} \mathrm{es:}$ | 0 d | 8 h | - 750 p |
| 276 cycl es: | - 16 d | - 14 h | 2100 p |
| 8 el apsed yrs | $+2 \mathrm{~d}$ | - 12 h | + 747 p |
| Tot al : | -14 d | 26 h | - 1353 p |

All you need do now is add the bench mark. The Hebrew cal endar is usually behi nd the Roman, so the total will be mostly negative numbers. The parts or hours COULD be positive in some problens. Wat ch your SCRI BAL ACCURACY!

| Sept 36 | 23 h | 204 p |
| :---: | :---: | :---: |
| -14 d | -26 h | - 1353 p |

Bor rowing:

| Sept |  | 45 h | 2364 p |
| :---: | :---: | :---: | :---: |
|  | - 14 d | -26 h | 1353 p |
| Sept | 21 | 19 h | 1011 p |

Wenever you are making a cal cul ation, it will be faster to use the conbi ned table instead of the indivi dual $t$ abl es of intercal ary years and el apsed years. But you can take either option.

If you find that cal cul ations of the day of the month are different from those of the day of the week, be sure that you used the right table for the right number!
4. 3 Cal cul ate with the use of the chart the day of the month of mol ad Ti shri for 1520 AD. The answer is bel ow.


Fromthe chart, part ll:


You can set up both the day of the week and the day of the month cal culations side by si de to save space if you like.
4. 4 Test yoursel $f$ on a compl ete cal cul ation of the mol Tishri, day of the week and the Roman date, for 1000 AD. Consult the chart in an efficient manner.

1000 AD 3761 BC

4761; 4760 years

Day of the week

| 200 cycl es: 50 cycl es | 5 1 1 $d$ |  | 200 p |
| :---: | :---: | :---: | :---: |
| 10 el apsed yrs. | 6 d |  | 339 |
| bench mark: | 1 d | 23 h | 204 |
| Tot | 13 d | 62 h | 1333 |
| Reduced: | 1 d | 15 h | 253 |


| 250 cycl es |  |
| :---: | :---: |
| 19 / | 4760 |
|  | XXX |
|  | 10 el apsed |

Day of the Roman month


The hours and the parts agree.
In 1000 AD the mol ad Tishri was Sunday, Sept. 1, 15 h 253 p.
By using the Hebrew cal endar chart, notice how much I ess space on the page a compl ete cal cul ation of the mol Ti shri now takes. And easi er, isn't it!

THE HEBREW CALENDAR
Tabl es of Reduced Days, Hours, and Parts for 19 Year Ti me Cycles, I ntercal ary Years, and Common Years.
I. Advancement of the Mbl ad Over a Full Number of Weeks

19 YEAR TI ME CYCLES



|  | Kossey |  |  |
| ---: | ---: | ---: | ---: |
| 8 | 6 | 22 | 528 |
| 9 | 4 | 7 | 324 |
| 10 | 1 | 16 | 120 |
| 11 | 6 | 0 | 996 |
| 12 | 3 | 9 | 792 |

II. Ti me Differences

19 YEAR TI ME CYCLES

| El apsed Cycl es | Time Difference |  |  |
| :---: | :---: | :---: | :---: |
| 1 | - 0d | - 1h | - 485p |
| 2 | - 0 | - 2 | - 970 |
| 3 | - 0 | - 4 | - 375 |
| 4 | - 0 | - 5 | - 860 |
| 5 | - 0 | - 7 | - 265 |
| 6 | - 0 | - 8 | - 750 |
| 7 | - 0 | - 10 | - 155 |
| 8 | - 0 | - 11 | - 640 |
| 9 | - 0 | - 13 | - 45 |
| 10 | - 0 | - 14 | - 530 |
| 20 | - 1 | - 4 | - 1060 |
| 30 | - 1 | - 19 | - 510 |
| 40 | - 2 | - 9 | - 1040 |
| 50 | - 3 | - 0 | - 490 |
| 60 | - 3 | - 14 | - 1020 |
| 70 | - 4 | - 5 | - 470 |
| 80 | - 4 | - 19 | - 1000 |
| 90 | - 5 | - 10 | - 450 |
| 100 | - 6 | - 0 | - 980 |
| 200 | - 12 | - 1 | - 880 |
| 300 | - 18 | - 2 | - 780 |

\#\#\#--------
I NTERCALARY YEARS

El apsed Inc.
Years

| 1 | $+18 d$ | $+15 h$ | $+589 p$ |
| :--- | :--- | :--- | :--- |
| 2 | +37 | +7 | +98 |
| 3 | +55 | +22 | +687 |
| 4 | +74 | +14 | +196 |
| 5 | +93 | +5 | +785 |
| 6 | +111 | +21 | +294 |
| 7 | +130 | +12 | +883 |

ELAPSED YEARS IN ONE 19 YEAR TI ME CYCLE

| El apsed Years | Bef ore 142 AD Ti me Difference |  |  |
| :---: | :---: | :---: | :---: |
| 1 | - 10d | - 21 h | - 204p |
| 2 | + 8 | - 6 | +385 |
| 3 | - 3 | - 3 | +181 |
| 4 | - 14 | 0 | - 23 |
| 5 | + 5 | - 9 | +566 |
| 6 | - 6 | - 6 | +362 |
| 7 | +13 | -15 | +951 |
| 8 | + 2 | - 12 | +747 |
| 9 | - 9 | - 9 | +543 |
| 10 | +10 | -17 | + 52 |
| 11 | - 1 | -15 | +928 |


| 142 AD and after Ti me Difference |  |  |
| :---: | :---: | :---: |
| -10d | - 21 h | - 204p |
| -21 | - 18 | - 408 |
| 3 | - 3 | +181 |
| -14 | 0 | - 23 |
| -24 | - 21 | - 227 |
| 6 | - 6 | +362 |
| -17 | - 3 | +158 |
| + 2 | - 12 | +747 |
| 9 | - 9 | +543 |
| 20 | - 6 | +339 |
|  | - 15 | +928 |

El apsed Years ------
---
1
2
3
4
5
6
7
8
9
10
11

11

|  | Kossey |  |  |  |  |  |  |
| :--- | ---: | ---: | :--- | :--- | ---: | :--- | :--- |
| 12 | -12 | -12 | +724 | -12 | -12 | +724 | 12 |
| 13 | +7 | -20 | +233 | -23 | -9 | +520 | 13 |
| 14 | -4 | -17 | +29 | -4 | -17 | +29 | 14 |
| 15 | -15 | -15 | +905 | -15 | -15 | +905 | 15 |
| 16 | +4 | -23 | +414 | -26 | -12 | +701 | 16 |
| 17 | -7 | -20 | +210 | -7 | -20 | +210 | 17 |
| 18 | +11 | -5 | +799 | -18 | -17 | +6 | 18 |

COMMDN YEARS

El apsed Comm
Years
1
2
3
4
5
6
7
8
9
10
11
12

Ti me Difference

| 10d | - 21h | 204p |
| :---: | :---: | :---: |
| - 21 | - 18 | - 408 |
| - 32 | - 15 | - 612 |
| - 43 | - 12 | - 816 |
| 54 | - 9 | - 1020 |
| 65 | - 7 | - 144 |
| - 76 | - 4 | - 348 |
| - 87 | - 1 | - 552 |
| - 97 | - 22 | 756 |
| - 108 | - 19 | 960 |
| - 119 | - 17 | - 84 |
| - 130 | - 14 | 288 |

\#\#\#------- -
as they are often different; e. g. 1 positive and 2 negative.

> - si gnifi es that the Hebrew cal endar is behi nd, or trails, the Julian cal endar.
> + si gnifi es that the Hebrew cal endar is ahead of, or l eads, the Julian cal endar.

PROGRAM V
MAKI NG ROMAN LEAP YEAR AND J ULI AN / GREGORI AN CORRECTI ONS

## PERFORMANCE GOAL 5

For any requi red Roman year, $A D$ or $B C$, you will correctly make the necessary adj ust ments for:
a) the Roman leap year / common year pattern
b) the conversion from the Julian to the Gregorian cal endar

In your day of the month cal culations of the mol Tishri.
By activel y working through the first four prograns, you' ve I earned how to cal cul ate the mol ad Ti shri for the specified years. Now you will appl y two corrections to the DAY OF THE MDNTH cal cul ations. Thi s will ext end your ability to find the mol ad of Tishri to virtually any Roman year requi red, whet her $A D$ or $B C$. Keep in mind that these corrections affect ONLY the day of the month cal cul ations! The day of the week part of your work needs no adj ustment.

The first correction involves the length of a Roman year. On a cal endar, a Roman year has either 365 days (common year) or 366 days (l eap year). In your cal cul ations for the day of the month, you used the average length of a Roman year, 365 1/4 days. How do you make al l owance for the difference?


#### Abstract

Kossey You add si $x$ hours to the mol ad (day of the month) for every year after a Roman leap year:


If the requi red Roman year is leap, add 0 hours.
If the requi red Roman year is one year after a leap year, add
6 hours.
If the requi red Roman year is two years after a leap year, add 12 hours.

If the requi red Roman year is three years after a leap year, add 18 hours.
(Don't become conf used by what a "l eap year" means. A Hebrew I eap year has 13 months; a Roman leap year has 366 days. The context will tell you what applies.)

How can you know if the requi red Roman year is leap or common? Si mply di vi de the year by four and not e the remai nder: 10 AD di vi ded by 4 gives a remai nder of two; 51 BC di vi ded by 4 gi ves a remai nder of three; 1977 AD di vi ded by 4 leaves a remai nder of one.

For AD years, the remai nder numerically corresponds to the Roman l eap year/commn year pattern:

A remai nder of 0 signifies that the required Roman year is leap. " 0 si gni fi
1
2
3 one after a leap. two after a leap. three after a leap.
$B C$ years have a different pattern of remai nders. Si nce 4 AD is a leap year, 1 AD is three years bef ore a leap year. 1 BC is a leap year, too, being four years bef ore 4 AD. Four years bef ore 1 BC is 5 BC, a leap year. And 9 BC is a leap year. Then 8 BC is one year after a Leap year; $7 B C$ is two years after; 6 BC is three after. If you di vi de these years by 4 , you can find the remai nder that corresponds to a leap year ( 9 di vi ded by 4 l eaves a remai nder of 1), a year after a l eap year, etc. This pattern of remai nders from $9 B C$ to $6 B C$ will be val id for years further back into antiquity.

The chart bel ow summarizes the remai nder patterns for both BC and AD Roman years.

Roman leap year corrections (di vi ding the requi red year by 4)

## AD:

If the remai nder is 0 , add no hours to the day of month cal cul ations

| $" 1$ | 1 | 6 hours |
| ---: | ---: | ---: |
| $"$ | 2 | 12 hours |

$$
\text { " } \quad 3 \quad 18 \text { hours }
$$

BC:
If the remai nder is 1, add no hours to the day of month cal cul ations

$$
\begin{array}{llr}
" & 0 & 6 \text { hours } \\
" & 3 & 12 \text { hours } \\
" & 2 & 18 \text { hour s }
\end{array}
$$

To insure that you can apply this correction to the day of the month, i ndi cate how many hours you would add for each of the following years.
5. 11520 AD
5. $10 \quad 1917$ BC
5. 21974 AD
5. 114 BC
5. 31983 AD
5. 12971 BC
5. 41871 AD
5. $13 \quad 1181 \mathrm{BC}$
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| 5. 5 | 1699 AD | 5. 14 | 1020 BC |
| :---: | :---: | :---: | :---: |
| 5. 6 | 31 AD | 5. 15 | 1984 AD |
| 5. 7 | 33 AD | 5. 16 | 1020 AD |
| 5. 8 | 142 AD | 5. 17 | 2001 BC |
| 5. 9 | 1486 BC | 5. 18 | 2001 AD |

The answers appear bel ow.

| 5. 1 | 0 | hours | 5. 7 | 6 | hours | 5. 13 | 0 | hours |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5. 2 | 12 | hours | 5. 8 | 12 | hours | 5. 14 | 6 | hours |
| 5. 3 | 18 | hours | 5. 9 | 18 | hours | 5. 15 | 0 | hours |
| 5. 4 | 18 | hours | 5. 10 | 0 | hours | 5. 16 |  | hours |
| 5. 5 | 18 | hours | 5. 11 | 6 | hours | 5. 17 |  | hours |
| 5. 6 | 18 | hours | 5. 12 | 12 | hours | 5. 18 |  | hours |

When you are checking your cal cul ations for the mol ad of Tishri, you may find that the day of the month calcul ation differs from the day of the week by six hours, twel ve hours, or ei ght een hours. Here's what has happened: You forgot to make the Romal leap year correction in the day of the month calcul ation! Di vi de the Roman year by 4 and inspect the remai nder on EVERY cal cul at i on invol ving the day of the mont h.
\#\#\#------ - -
The second correction to the day of the month cal cul ation of the mol ad Tishri concerns the corresponding Gregorian cal endar date of a day on the Jul ian cal endar. By understanding the reasons for the change from the Julian cal endar to the Gregorian, you will have no difficulty in performing this correction.

The Julian cal endar was named after Julius Caesar; the Gregorian cal endar after Pope Gregory XIII. Both cal endars are "Roman. " The Gregori an cal endar is what you use every day.

When did the Roman cal endar, based upon an aver age yearly length of 365 1/ 4 days, come into use? The answer is 45 BC. Wth the ai d of the Egyptian astronomer Sosi genes, Juli us Caesar compl et el y revi sed the previ ous I uni sol ar cal endar, whi ch had drifted badly with respect to the seasons. To effect the reform "46 BC" had 445 days assigned to it in order to correct for all the previ ous errors. That year was very appropri at el y called the "year of conf usi on!" In 45 BC the ver nal ( spring) equi nox occurred on March 25.

The Julian cal endar wasn't without its faults, however. The aver age Jul i an year was el even mi nutes and fourteen seconds LONGER than a "tropi cal year." (A tropical year is measured from one vernal equi nox to the next.) After 128 years the Julian cal endar had an extra day, compared to an equal number of tropical years.

By the time of the famous Council of Ni cea in 325 AD, the Juli an cal endar was about three days behi nd the tropical year. Thi s meant that the vernal equi nox was sever al days "early" on the Jul $i$ an cal endar. Accordingly, the chur chmen based thei $r$ rules for the date of Easter on the spring equi nox falling on March 21 of the Julian cal endar. It had been March 25 in 45 BC.

Towards the end of the Count er-Ref ormation in the si xt eent h cent ury, the spring equi nox had "crept back" to about March 11. To al leviate the problem Pope Gregory ref ormed the Jul i an cal endar by del eting ten days from the month of Oct ober 1582: The day after Oct ober 4 officially became Oct ober 15. So after 1582 the spring equi nox shifted back to March 21--where it had been during the time of Constantine the Great. ( Of course, the equi nox never shifted; the

Pope Gregory al so i nvoked a new rul e concerning Roman I eap years. The Jul i an cal endar consi ders every fourth year as having 366 days. To shorten the new Roman cal endar by three days in 400 years, the Pope decl ared that cent ury years NOT EVENLY DI VI SI BLE BY 400 woul d remai $n$ COMMDN (no February 29).

Ther ef ore, 1700 AD, 1800 AD, and 1900 AD, whi ch are not evenl y di vi si bl e by 400, were common years according to the Gregorian cal endar rules. By the old Julian cal endar, they woul d have been leap years.

Instead of the el even minte error in the aver age Julian year, the average Gregorian year is only 26 seconds toolong.

How do you convert a Julian date into a day on the Gregorian cal endar? J ust add the TOTAL DAYS that have been dropped from the Jul i an! Bef ore 1582 AD no correction is needed, as alp the Roman dat es are understood to be Julian. But after 1582 AD, you must add at least ten days to the Julian day of the month cal cul ation for the mod Ti shri.

Bear in mind that 1600 AD was a leap year in both the Jul i an and the Gregori an cal endars. The di fference bet ween the cal endars remai ned ten days until 1700. The Gregori an cal endar omitted February 29 that year (because the year wasn't evenl y di vi si ble by 400), while the Jul i an ret ai ned the extra day. From 1700 to 1799, the juli an cal endar was el even days behi nd the Gregorian si nce that was the tot al number of days dropped.

In the same manner, from 1800 to 1899 you must add twel ve days to the Julian date to find the corresponding Gregorian day. During the $1900^{\prime} \mathrm{s}$ and 2000's, you add thirteen days.

How many days will you add to your day of the month cal cul ations of the mol ad of Tishri to convert from the Jul i an cal endar to the Gregorian cal endar for each of the following years? (Bef ore 1582 AD, si mpl y respond " 0 d".)
5. 191520 AD
5. 201601 AD
5. 211798 AD
5. 221914 AD
5. 231851 AD
5. 241666 AD
5. 251583 AD
5. 261984 AD
5. 272001 AD
5. 282145 AD
5. 291979 AD
5. 301712 AD

The answers appear bel ow.
5. $19+0 \mathrm{~d}$
5. $25+10 \mathrm{~d}$
5. $20+10 \mathrm{~d}$
5. $26+13 \mathrm{~d}$

| $5.21+11 d$ | $5.27+13 d$ |
| :--- | :--- |
| $5.22+13 d$ | $5.28+14 d$ |
| $5.23+12 d$ | $5.29+13 d$ |
| $5.24+10 d$ | $5.30+11 d$ |

Nei ther of the corrections to the day of the month cal cul at i on the mol ad Tishri is compl icated. However, you must remember to perform them whenever the Roman year calls for such adj ust ments. The I ast problemin this programwill challenge you to put toget her all the cal endar skills you have learned thus far.
5. 31 What is the day of the week and the date of the Roman mont h for the mol ad Tishri in 2055 AD? (You may use the chart of reduced numbers.) This probl em is worked out in detail for you on the next page.

What is the day of the week and the date of the Roman month for the mol ad of Tishri in 2055 AD?
A. The ELAPSED TI ME is: - 3761

- 2055
- 5816 - subtract 1 goi ng from $B C$ to $A D$ : 5815 years.
B. 5815 years is 5815 / 19 ni net een year cycl es.

C. THE DAY OF THE WEEK FOR THE MOLAD TI SHRI :

From the tables, find the advancement of the mol ad.

| 300 6 1 | 19-year cycl es: 19- year cycl es: year of next cycle: | $\begin{array}{ll}1 \mathrm{~d} \\ 2 & d \\ 4 & d\end{array}$ | 21 h 3 h 8 h | $\begin{aligned} & 300 \\ & 330 \\ & 876 \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Reducing this: |  | 7 d | $\begin{aligned} & 32 \mathrm{~h} \\ & +1 \end{aligned}$ | $\begin{aligned} & 1506 \\ & 1080 \end{aligned}$ |  |
|  |  | $\begin{array}{r} 7 d \\ +1 d \end{array}$ | $\begin{array}{r} 33 \mathrm{~h} \\ -24 \mathrm{~h} \end{array}$ | 426 |  |
|  | The advancement is: | 1 d | 9 h | 426 | over full of weeks. |
| Add the advancement the bench mark: |  | + 1 d | 23 h | 204 |  |
|  |  | 2 d | 32 h | 630 |  |
|  | This reduces to: | 3 d | 8 h | 630 |  |

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## D. THE DAY OF THE ROMAN MDNTH:

From the tables, find how far behi nd the Roman cal endar the Hebrew cal endar is:

"Borrow' 2 x 1080 parts, and one day:
Sept. 3545 h 2364 p
Add the time difference in the cal endars to the benchnark:


Two corrections must be made, that of the ROMAN I eap year, and the Julian / Gregorian:

$$
513
$$

4 / 2055
20
$5 \quad 3$ years after a Roman leap year means
4 you mast add 18 HOURS.

15
12
$-3$
During the 1900's, the correction to the Julian cal endar is 13 days. Si nce 2000 is evenl y di vi si ble by 400, the correction during the 21st cent ury is STI LL 13 days.


Roman leap year correction Julian / Gregorian correction 13 d

Mbl ad Tishri: Sept. 218 h 630 p parts and hours agree with day of week

PROGRAM VI

## APPLYI NG THE POSTPONEMENT RULES TO FI ND TI SHRI ONE

PERFORMANCE GOAL 6A: Gi ven the mol ad Ti shri for a requi red year, you will correctly determine the day of the week and the day of the month of the Feast of Trumpets, Tishri 1, by applying from memory the four post ponement rules expl ai ned in this program

Example: In 1987, the mol ad of Tishri will be:
Sept ember 23 Vednesday ( 4 d) 3 h 77 p
What is the date for the Feast of Trumpets?

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Answer: Septenber 24 Thursday (by rule two)
PERFORMANCE GOAL 6B: Without consulting notes, you will correctly expl ain (in a bri ef paragraph for each rule) why each postponement for Ti shri 1 is important for the Hebrew cal endar.

PERFORMANCE GOAL 6A: After correctly cal cul ating both the day of the week and the day of the month for the mol ad of Tishri the next step is to find the date of Ti shri 1, the Feast of Trumpets. You will anal yze the time of the mol ad to see how four post ponement rules affect the Feast of Trumpets.

At the outset, you should understand that a conj unction of the earth, mon, and sun takes place compl et el y apart from man's doi ngs. The MOLAD CANNOT BE POSTPONED BY HUMAN ENACTMENTS! Ti shri 1, the ci vil New Year in the Hebrew cal endar, is what is post poned.

Here are the four post ponement rules, prefaced by a general statement of the case where there is no postponement:

Wen the mol ad Ti shri occurs at a time of the week that's unaffect ed by the four post ponement rules, the Feast of Trumpets is on the SAME DAY as the mol ad.

Rule one: When the mol ad of Ti shri occurs AT NOON OR LATER ( 12 h 0 p or more in your cal cul ations), the Feast of Trumpets is post poned until the next day.

Rul e two: When the mol ad of Tishri OR a post ponement occurs on a Sunday, Wednesday, or Fri day, the Feast of Trumpets is post poned one day, to a Mbnday, Thursday, or Sabbath, respectively.

Rul e three: When the mol ad of Ti shri of a COMMDN YEAR is on a Tuesday, at or after ( 3 d ) 3 h 204 p the Feast of Trumpets is post poned to a Wednesday, and by Rul e two, further post poned to a Thursday.

RULE FOUR: When the mol ad of Ti shri of a COMMON year I MMEDI ATELY FOLLOW NG an int ercal ary year occurs on a Mbnday, at or after (2 d) 9 h 589 p the Feast of Trumpets is postponed to a TUESDAY.

You should take notice that the maximpost ponement of Tishri 1 is two days. Al so real ize that the order of these rul es is si gni ficant. They are easi est to appl y in the numerical order above. Rule one governs all afternoon time periods, so rules three and four ONLY AFFECT A PORTI ON OF THE MDRNI NGS.

You will qui ckly discover that rules one and two are by far the most frequent ly used, both i ndi vi dually and toget her -- one, two, or one and two. And years where no post ponement of Ti shri 1 is required are rather common, too.

Thi s means, of course, that rules three and four are quite infrequent, especially the last one. Rule three has rule two "built in", since it is a very specific case. Ther ef ore, rule three and rule four are al ways separatel y used. Whenever rul es one and two apply, ei ther separat el y or in combi nation, you know in advance that rul es three and four will not be i nvol ved. All this speeds up your use of these rules consi derably.

Bef ore you see examples of each rule applied, one more thing needs to be expl ai ned. How do you know when the year in questi on is common, and when does it immedi at el y follow a leap year?

Remember how you det ermined el apsed ti me by adding (or subtracting) the required year from the bench mark? Well, the year in question is the year AFTER the number of el apsed years. The di stinction bet ween the terns "el apsed years" and "required year" is quite similar
to your age. All through your 21st year of life, your age is twenty!
What do you do to find whi ch year of the cycle is a requi red year? Add 1 to the remai nder you al ready found when you di vi ded the el apsed years by 19. This new remai nder will be the number of the year in the cycle. If the remai nder for el apsed years is 10, the requi red year is year 11 of the 19 -year cycle.

You will recal that for the years 142 AD (handwritten note: 256 AD -- Dr. Hoeh correct date) and after, the intercal ary years are 3, $6,8,11,14,17,19$. The years immedi at ely following these years are the ones you are concer ned wi th for rule four: 4, 7, 9, 12, et c. Al I the years in the cycle whi ch aren't intercal ary are common.

Wenever you are working with years earlier than 142 AD, be sure to use the proper intercal ary years, whi ch are one year earlier than the present cycle: $2,5,7,10,13,16,18$. The year after these leap years will then apply for rule four.

Now you will learn how to anal yze the mol Ti shri to apply the post ponement rul es.

1492 AD: Mblad Ti shri was September 21, Friday 6 d 9 h 1011 p.
19 h 1011 p is after 12 h 0 p ; RULE ONE post pones to the Sabbath. The Sabbath is a permissible day; no further postponement.

Feast of Trumpets was September 22, Sabbath.
1584 AD: mol ad Ti shri was September 5, Wednesday 4 d 2 h 852 p.
2 h 852 p is before 12 h 0 p ; rul e one not invol ved. Wednesday is a forbi dden day; RULE TWD appl i es.

Feast of Trumpets was September 6, Thursday.
1615 AD: mol ad Ti shri was September 22, Tues day 3 d 20 h 804 p.
20 h 804 p is after 12 h 0 p ; RULE ONE post pones to Wednesday. Wednesday is a forbi dden day; RULE TVO re-postpones to Thursday.

Trumpets was September 24, Thursday.
1632 AD: mol ad Ti shri was September 14, Tuesday, 3 d 6 h 1014 p.
6 h 1014 p is bef ore 12 h 0 p. Rule one doesn't apply. Tuesday is a permissible day. Rule two doesn't apply. 6 h 1014 p is after 3 h 204 p. Rul e three could be invol ved. Is 1632 a common year?

The el apsed time from 3761 BC is 5392 years to 1632 AD. Then the year in question is the 5393rd year, or the 16th year of a cycle after 283 full cycles. This is a common year. RULE THREE does apply.

Ti shri 1 is September 16, Thursday.
841 AD: mol ad Ti shri was September 19, Mbnday, 2 d 11 h 735 p.
Rul es one, two, and three, don't apply. The mol ad is on a Mbnday, and 11 h 735 p is after 9 h 589 p . Is 841 AD a common year i medi at el y after a l eap year? From 3761 BC to 841 AD is 4601 years; the requi red year is the 4602 year. Thi s is 242 19-year cycl es pl us 4 years of the next. This is a common year immedi at el following a leap year. RULE FOUR applies.

Trumpets was Sept ember 20, Tues day.
1910 AD: mol ad Ti shri was October 4, Tuesday, 3 d 0 h 61 p.
0 h 61 p is bef ore 12 h 0 p ; r ule one doesn't apply. Tuesday is a permissible day; rule two doesn't apply. 0 h 61 p is before 3 h

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204 p; rule three doesn't apply. Rul efour can't apply to Tuesday. No post ponement.

Trumpets is October 4, Tuesday.
As you can see, only a few seconds are needed to check a mol ad of Ti shri for applicable post ponements. Don't under-estimate the i mportance of doing this oper at i on correctly, however! All of the ot her Hol y Days are derived from the date of Ti shri one!

Practice applying the post ponement rules to the following mol ads. The correct dates for the Feast of Trumpets are listed on the next page, al ong with the post ponement rules which are needed.

6a. 11264 AD September 22, Mbnday 2 d 13 h 351 p
6a. 21255 AD September 3, Friday 6 d 3 h 95 p
6a. 31259 AD September 18, Thurs. 5 d 15 h 865 p
6a. 42001 AD Septenber 17, Mbnday 2 d 22 h 106 p
6a. 52008 AD Septenber 30, Tuesday 3 d 1 h 1057 p
6a. 62014 AD September 24, Wednesday 4 d 8 h 339 p
6a. 71984 AD Septenber 25, Tuesday 3 d 11 h 976 p
6a. 81985 AD September 14, Sabbath 7 d 20 h 772 p
6a. 9462 AD September 10, Mbnday 2 d 1 h 511 p
6a. 10496 AD September 23, Mbnday 2 d 10 h 644 p
6a. 11134 AD October 5, Mbnday 2 d 23 h 343 p
6a. 12118 AD October 2, Sabbath 7 d 21 h 1009 p
6a. 13588 BC Septenber 27, Tuesday 3 d 3 h 209 p
6a. 141953 BC Sept enber 12, Sunday 1 d 12 h 4 p
Answers on following page:

Answer s:
6a. 1 Sept enber 23, 1264 AD; Tuesday 1
6a. 2 Septenber 4, 1255 AD; Sabbath 2
6a. 3 Sept ember 20, 1259 AD; Sabbath 1, 2
6a. 4 Sept ember 18, 2001 AD; Tuesday 1
6a. 5 September 30, 2008 AD; Tuesday
6a. 6 Sept ember 25, 2014 AD; Thursday
6a. 7 Sept ember 27, 1984 AD; Thursday
6a. 8 Sept ember 16, 1985 AD; Mbnday
6a. 9
6a. 10 Sept enber 24, 496 AD; Tuesday
6a. 11 October 6, 134 AD; Tuesday

Rule

2
3
1, 2

4
1

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| 6a. 12 | Oct ober 4, 118 AD; Mbnday | 1,2 |
| :--- | :--- | :--- | :--- |
| 6a. 13 | Sept ember 29, $588 \mathrm{BC} ;$ Thur sday | 3 |
| 6a. 14 | Sept ember 13, $953 \mathrm{BC} ;$ Mbnday | 1 |

6a. 14 Sept enber 13, 953 BC; Mbnday 1

PERFORMANCE GOAL 6B: You do not need to understand the reasons behi nd the post ponement rules in order to correctly apply them However, your appreci ation of the Sacred cal endar will be enhanced by grasping the purpose of each rule.

## WHY RULE ONE, NOON OR AFTER POSTPONEMENTS?

Whenever the mol ad of Tishri occurs at noon or after ( 12 h 0 p ) Ti shri one is post poned to the next day (at least): But how did the mol ad come to be associ ated with noon? Rule one points back to the initial formil ation of the Hebrew cal endar -- in the days of Seth, according to Josephus. God intended that the heavenl y bodi es would intrigue man to study their movements carefully. "Let there be lights in the firmment of the heaven to di vi de the day fromthe ni ght; and l et them be for signs, and for seasons, and for days, and for years" ( Gen. 1: 14).

As you may have observed, the sun, moon and stars rise and set each day. But the sun rises each day about four mi nutes later with respect to the stars. It has a proper motion of its own, independent of the stars. Thi s motion is not due to the daily rotation of the earth on its axis, whi ch gives the illusi on that the sun moves west ward each day. The appar ent path of the east ward jour ney of the sun through the stars, known as the "ecliptic", is due to the earth's annual orbital revol ution.

The earth is tilted 232 degrees on its axis. By studying the di agram bel ow, you can see that the pl ane of the earth's equat or is incli ned 232 degrees fromthe pl ane of the earth's orbit. Astronomers call the projection of the earth's equator into space the "cel estial equat or ".
( NOTE: To vi ew the di agram menti oned above, see the file HEB- CAL1. TIF in the I mages $\backslash$ Ot her WCG di rect ory.)

Seasons occur because the earth's orbital pl ane (the pl ane of the earth's orbit) is not coinci dent with the plane of the earth's equat or The vernal equi nox takes pl ace when the apparent northward movi ng sun crosses the cel estial equator. It agai n intersects the cel estial equat or when the sun passes bel ow the cel estial equat or moving southward--at the time of the aut umal equi nox. On the days of the equi noxes, the day and ni ght periods are equal.

Consider the next di agram which is a cl oseup of the vernal equi nox. The moment of the equi nox occurs when the center of the sun crosses the cel estial equat or. But the sun's apparent di ameter is $1 / 2$ degree; ther ef ore the TRAI LI NG EDGE of the sun is still $1 / 4$ degree behi nd the equi noctial point.
( NOTE: To vi ew the di agram menti oned above, see the file HEB- CAL2. TI F in the I mages \Ot her WCG di rect ory.)

How long will it take for the trailing edge of the sun to pass the

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point of the vernal equi nox? The sun's east ward progress al ong the ecliptic is about 1 degree per day (since the earth revol ves 360 degrees around the sun in about 365 days). In 24 hours the sun moves 1 degree; in 12 hours it moves $1 / 2$ degree; in 6 HOURS it moves $1 / 4$ degree.

Si $x$ hours before sunset is 12 noon. Unl ess the equi nox occurs BEFORE noon, the sun's trailing edge will not have passed the cel estial equat or by sunset! When the equi nox occurs at noon or later, Sol is still a wi nter sun. The first day of spring is ther ef ore assi gned to the following day.

This is one reason why noon became a logi cal demarcation poi nt for time in astronom cal matters. Noon is a stable observation poi nt for time, irrespective of the observer's latitude. (The time of sunrise and sunset, but not noon, vary consi derably during the course of a year, especially in the extreme latitude.) Fromearliest known times until 1925, astronomers had traditionally used noon as a reference poi nt for the day. Thus noon became in antiquity a limit point not only for the equi nox, but al so for the mol ad.

Just as noon arbitrated the first day of spring and the first day of fall, it served a similar function with the assigned day of the mol ad -- that is, the mol ad or conj unction of the mon had to have a natural and arbitrary limit, in this case noon.

## WHY RULE TWO, FORBI DDEN DAYS?

Thi s post ponement rule prevents Holy Days fromfalling on a Sunday (during the fall) and the Passover occurring at awkward times:

If Trumpets could occur on a Wednesday, the Day of At onement, Tishri 10, would fall on a Friday, the preparation day for the Sabbath! And the Passover would al so be observed Sat urday ni ght, a most difficult time.

If Trumpets could occur on a Friday, the Day of At onement would be on a Sunday, the day after the Sabbath.

If Trumpets could occur on a Sunday, then the first day of the Feast of Tabernacles, as well as the last Great Day would be on a Sunday.

WHY RULE THREE, THE TUESDAY-COMMDN YEAR POSTPONEMENT?
As you recall, the maxi mum length of a common year in the Hebrew cal endar is 355 days. Without this rule, a common year migh have 356 days!

Anytime in the morning of a Tuesday, 3h 204 pis affected by this rule. Why this particular moment?


Thi s number is the same as the seventh day 12 hours and no parts. This means that the next mol ad Ti shri would occur on a Sabbat h at noon. But rule one says you must post pone to a Sunday, and rule two says to re-postpone until a Mbnday.

From Tuesday until Mbnday is six days. The full number of weeks in a common year (50) gi ves $7 \times 50=350$ days. So that year woul d have 356 days -- if rule three didn't intervene.

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The mi nimm number of days an intercal ary year can have is 383 days. If rule four wer en't in effect, some leap years would have only 382 days. An aver age leap year has 5 d 21 h 589 p over a full number of weeks, whi ch is 54.

Suppose that a common year just after a leap year began on the same day as the mol ad, Mbnday 2 d 9 h 589 p . When woul d the preceding l eap year have begun?

## Mbnday

 $\begin{array}{rlrlll}2 & d & 9 & h & 589 & p \\ -(5 & d & h & 589 & \text { p) excess over full number of }\end{array}$ ----------------- weeks. - $3 \mathrm{~d}-12 \mathrm{~h}$$-3 \mathrm{~d}-12 \mathrm{~h}$ is the same as $-4 \mathrm{~d}+12 \mathrm{~h}$. Whenever you are working with negat ive days of the week, you merely add 7 d to find out what day of the week you are actually on. $-4 \mathrm{~d}+7 \mathrm{~d}$ is the 3 d of the week, or Tuesday. The mol ad of the leap year thus occurred on a Tuesday at noon. But rule one says that this must be post poned to a Wednesday, and by rule two, re-post poned to a Thursday.

If the leap year began on a Thursday and ended on a Mbnday, there would be four days over the full number of weeks. $7 \times 54$ weeks is 378 days. 378 days plus 4 days is onl y 382 days; less than the mi mum

Wthout these post ponement laws, the sacred cal endar would be in a perpetual state of conf usion. Holy Days would fall on a Sunday. The lengths of years would be irregul ar. Cal endar reformers would be tempted to tamper with the sacred cal endar more often. Pi cture the difficulty of a deacon trying to keep a Sabbath Day hol y while frantically making last mute preparations for the Passover ceremony!

But all that turmil is avoi ded by four very si mple and easily appl i ed post ponement rules. I nstead of the Sacred Festivals being subordi nate to the Hebrew cal endar the latter serves the Holy Days.

PROGRAM VI I

## COUNTI NG THE DAYS OF THE WEEK AND THE DAYS OF THE MDNTH

## PERFORMANCE GOAL 7A:

You will count forward or backward a specified length of time from a gi ven day of the week and correctly determ ne the new day of the week.

For exampl e:
What day of the week is 164 days bef ore Thursday?
Answer: Monday.

PERFORMANCE GOAL 7B:
Gi ven the date and the day of the week for any one day in a Roman or Hebrew month, you will correctly i ndi cate:
a) The day of the week for any ot her gi ven date in that month, or
b) the dates for any specified day of the week in that month.

For exampl e:
a) If the 13th of a month is on a Friday, what day of the week woul d the $4 t h$ be?

Answer: Wednesday.
b) What are the dates for Tuesday in a month when a Monday in that month is on the 9th?

Answer: 3, 10, 17, 24, (31).

## COUNTI NG THE DAYS OF THE WEEKS 7A

Counting the days of the week is so si mple that you' re probably inclined to use "fi nger-cal cul ati ons" quite of ten! Despite the apparent lack of complexity, however, mistakes can easily creep in. Wbuldn't you prefer a less embar rassing techni que than finger-counting to expl ai $n$ why a Hol y Day occurs when it does? A more dignified demonstration is possi ble!

All of the Sacred Festivals of God are anchored to Ti shri one. For example, the Day of At onement is ten days after Trumpets. Passover is 164 days bef ore. Accurate det er mi nation of the day of the week of $t$ he Festival s depends on correct counting procedures. Basic arithmetic is all that's needed.

As you recall from Program 3, Cal cul ating the day of the week of the mol ad Tishri, multiples of full weeks do not affect the final day of the week i nvol ved. Onl y the excess, or remai nder, is si gni ficant. Just as seven days from Mbnday is still a Mbnday, 147 days (twenty one weeks) bef ore or after Mbnday is still a Mbnday.

In the same manner as Program 3, equate the days of the week with the numbers 1 to 7. Sunday is the first day, Wednesday is the fourth day, the Sabbath is the seventh day.

Days AFTER a ref er ence day are ADDED to the numeri cal equi val ent of the ref erence day. Days BEFORE the ref er ence day are SUBTRACTED from the numerical equival ent. You will find it much easier if you take out full multiples of weeks bef ore you add or subtract.

Here are some examples. On what day of the week will At onement occur when Trumpets is on a Mbnday? At onement (Ti shri 10) is ni ne days after Trumpets, or one week and two days later. Mbnday is the second day of the week.
$2+2=4$ The fourth day of the week is Wednesday.
On what day of the week will Passover occur when Trumpet sis on a Sabbath? Passover is 164 days bef ore Trumpets. Di vi de 164 by 7, and you can express the time interval as 23 weeks and 3 days.
$7{ }^{-}$- $3=4$ The fourth day is Wednesday. (It's observed Tuesday ni ght.)

What happens when you want to find the day of Passover if Trumpets were on a Mbnday? Arithmet ically, the probl em is simply $2-3=-1$. What day of the week does this negative number indi cate? Si mply ADD 7 to any negative number in these cal culations of the day of the week:

$$
-1+7=6 \text { The si xth day of the week is Friday. }
$$

Here is an alternative way of expressing the same reasoning. First indicate the tot time in terns of full weeks and days over a ful week As an illustration, Passover is al ways 164 days bef ore $\operatorname{Tr}$ umpets. This is the same as 23 weeks and 3 days before. If Trumpets occurs on Tuesday, 23 weeks bef ore the Tuesday of Trumpets is another Tuesday. What is three days bef ore Tuesday? One day bef ore is Mbnday; t wo days bef ore is Sunday; three days bef ore Tuesday is the Sabbath. Passover is on a Sabbath, observed Friday eveni ng.

Pent ecost al ways occurs on a Sunday because of incl usi ve reckoni ng. Leviticus 23: 15 states that Pentecost is count ed "from [ begi nni ng with] the morrow after the Sabbath, from the day that ye
brought the sheaf of the wave of fering ..." The wavesheaf offering was performed on Sunday, the day after the weekly Sabbath. With a Sunday the first day, seven full weeks ( or forty-ni ne days) takes one to the fiftieth day, again on a Sunday.

By counting incl usi vel y, 50 days is seven weeks. If you di vi de seven weeks by seven, you have no remai nder. Sunday is the first day of the week, so arithmetically you have $1+0=1$, or Sunday, as you started.

Practice finding the day of the week in the following problens. Set them up in terns of a simple arithmetical expression.

7a. 1 What is 17 days after a Mbnday?
7a. 2 What day is 6 days bef ore Tuesday?
7a. 3 What is 50 days after a Sat urday.
7a. 4 On what day does Passover occur if Trumpets is on a Tuesday?
7a. 5 What day is 53 days bef ore Thur sday?
7a. 6 If Passover is on a Fri day (Thursday evening), when does At onement occur?

The answers are on the next page.

7a. 117 days $=2$ weeks, 3 days. Mbnday is the second day of the week. $2+3=5$; Thursday.
7a. 2 Tuesday is the third day of the week. $3-6=-3$. Add 7 to change this to a real day: $-3+7=4$; Wednesday

7a. 350 days $=7$ weeks, one day. Sabbath is the seventh day. $7+1=$ 8. Di vi de by seven and look at the remai nder: 1; Sunday.
7a. 4164 days $=23$ weeks, three days. Tuesday is the third day. $3-3$ $=0$. Convert this to a real day by adding seven: $0+7=7$; Sabbath.

7a. 553 days is 7 weeks, 4 days. Thursday is the fifth day. $5-4=1$; Sunday.

7a. 6 Passover and Trumpets are separ at ed by 23 weeks, 3 days.
At onement is ni ne days after Trumpets, or one week, two days later. Passover is on the sixth day. Then Trumpets is on:

When $6+3=9$. El im nate the seven (9-7): 2; Mbnday.
When Trumpets i s on Monday, At onement will be: $2+2=4 ;$ Wednesday.

## COUNTI NG THE DAYS OF THE MDNTH, 7B

Hebrew mont hs have ei ther 29 or 30 days. Mbst mont hs on a Roman cal endar contain 30 or 31 days. As you recall from the form of most printed cal endars, the same days of the week are pl aced over top one anot her.

For example, take a month of 30 days whose first day is Sunday. It will appear:

| Sun | Mbn | Tue | Wed | Thu | Fri | Sat |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 |


|  |  |  | Kossey |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 22 | 23 | 24 | 25 | 26 | 27 |
| 29 | 30 |  |  |  |  |

Take particular notice of the sequence in the col ums (up and down the page). The Tuesday col um cont ai ns al the days of the month whi ch fall on Tuesday. They are, in thi s case, 3, 10, 17, 24.

What number separates these days of the month? Seven. One week from the third day of the month is 7 days later, or $3+7$ days into the month. One week after the 10 th i s $10+7$, or the 17 th , etc. Learn to count by sevens for numbers up to 31.

Not every month will begin with the first day of the week, of course. You still can utilize the sequence of numbers separ at ed by seven no matter what day of the week the month commences.

7b. 1 What is the sequence of days of the month which occur on the Sabbath when the first Sabbath is on the 2nd?

7b. 2 What is the sequence of days of the month that fall on Mbnday when the first day of the month is a Friday?
7b. 3 If one of the Tuesdays of the month is the 22 nd, what would be the ot her days of the month for Tuesdays?

7b. 4 What are the days of the month for the Sabbath in a month where one of themis on the 16th?

7b. 5 The month of $A B$ has 30 days. The $10 t h$ of $A b$ is a Thursday. On what days of the month will the Mbndays occur?

7b. 6 The month of Thammz has 29 days. The $6 t h$ of Ab is a Sabbath. On what days of the month will the Vednesdays occur?
7b. 7 If the 14 th of a month is on Wednesday, what day of the week woul d the 26th be?

7b. 8 If the 22nd of the month is on Sunday, what day of the week woul d
the 10 th be?
The answers to these problems are bel ow.
7b. 1 2, 9, 16, 23, 30 are Sabbaths.
7b. 2 Friday, the $6 t h$ day of the week, is the first of the month. Then Mbnday, three days after Friday, is the fourth. The Mbndays are: 4, 11, 18, 25.

7b. 3 The 29th is the last Tuesday in the month. To find the others, count backward "by sevens" fromthe 22nd: 15, 8, 1.

7b. 4 16, 23, 30; 9, 2. These are the Sabbat hs.
7b. 5 10th = Thursday; Mbnday is 4 days later, or the 14 th. 7, 14, 21 , 28 are the Mbndays.

7b. 6 Sabbath $=6 t h$ of the month. Wednesday is three days earlier in the week, or the 3rd: The Wednesdays in Thammuz (in thi s case) are 3, 10, 17, 24.

7b. 7 Two weeks after the 14 th is the 28th, a Vednesday. The 26th is t wo days before the 28, so it is a Mbnday.

7b. 8 Two weeks bef ore the 22 nd is the $8 t h$, a Sunday. The 10th will be t wo days later, or Tuesday.

# Kossey <br> DETERM NI NG THE DATES OF THE ANNUAL FESTI VALS 

PERFORMANCE GOAL 8:
Given the day of the week and the Roman date for the Feast of Trumpets, you will correctly determine the day of the week and the day of the Roman month for each of the ot her annual Festivals in the Roman year. These Festivals are listed in Leviticus 23.

One of the most practical aspects of studying the Hebrew cal endar is understanding the layout of Sacred Festivals. Obvi ously, if the Western Wbrld were using the Hebrew cal endar for routine busi ness affairs, there would be no need for transforming the dates of Trumpets, At onement, Passover, etc., to the Roman cal endar. But si nce this isn't the case, one must know what day of the Roman cal endar each Hol y Day corresponds.

Program 8 builds upon the cal cul ation skills you have I earned in program 7. Take time to revi ew the previous program as is necessary for you.

In order to understand the rel ationshi $p$ of the dates of $t$ he $\mathrm{Hol} y$ Days, I et's summarize the cal endrical information given in Leviticus 23 Ref er ence Festival Hebr ew Dat e

| Lev. | $\begin{aligned} \hline 23 & : 5 \\ : & 6-7 \\ & : 8 \\ & : 15-16 \end{aligned}$ | Passover <br> 1st day of <br> Unl eavened Br ead <br> 7th day of <br> Unl eavened Bread Pent ecost | Ni san 14; 164 days bef ore Ti shri 1 Ni san 15 <br> Ni san 21 <br> 50 days begi nni ng with the Sunday of the wavesheaf of fering, (which is the day after the [a] regul ar weekly Sabbat h) [ NOT AN ANNUAL SABBATH ] [ THE SUNDAY WH CH IS ] during the Days of Unl eavened Bread. (Al ways a Sunday.) <br> [ See the WKN 5-11-87 \& GN 6-74 for comments in brackets ] |
| :---: | :---: | :---: | :---: |
|  | 24 | Tr umpet s | Ti shri 1 |
|  | : 27 | At onement | Ti shri 10 (9 days after Trumpets) |
|  | : 34-35 | 1st day of |  |
|  |  | Taber nacl es | Tishri 15 |
|  | : 36 | Last Great Day | Ti shri 22; (the "8th day" of the Festival, or 7 days after Ti shri 15) |

One of the first questions you will have is this: How do you know that Ni san 14, Passover, is 164 days bef ore Tishri 1? The answer is quite si mple: the intervening mont hs are al ways the same length, no matter whet her the year is l eap or commo. Ni san has 30 days; $\mathrm{Zi} f$ (I yar) 29; Si van 30; Tammuz 29; Ab 30; El ul 29. Then comes Ti shri. After the 14 th day, Ni san has 16 days remai ni ng in the month. The next five mont hs, I yar through El ul have 147 days $(29+30+29+30+29)$. One more day to Tishri 1. Add $16+147+1$, and you have 164 days.

Remember the "count by seven" pattern that separ at es the same day of the week in any month? You I earned in Program 7 that days 1, 8, 15, 22, and 29 of a month are the same day of the week. Notice that three of the four Hol y Days in the fall season occur on the SAME DAY OF THE WEEK!

Al l the fall Holy Days occur in the month of Tishri, on the 1st, 10th, 15th, and 22nd. Once you det ermine the day of the week of Trumpets, you know i mredi at el y the day of the week of the first day of Tabernacl es and the Last Great Day. They're all the same! And At onement
i s simply two days later in the week than Trumpets.
Another conveni ent feat ure of the Holy Days is that Tishri 1 can fall only on a Mbnday, Tuesday, Thursday or Sabbath. (Recal I Post ponement Rul e 2 in Program6?) This means that in any Roman year, the Holy Days will have one of these four patterns based upon what day of the week the Feast of Trumpets occurs. You have a "Mbnday pattern", a "Tuesday pattern", a "Thursday pattern", and a "Sabbath pattern". Once you understand these four patterns -- and they need NOT be memorized, as you will see! -- you can list the day of the week of ALL the Sacred Festivals for any gi ven year in about 30 seconds. Yes, it's that si mpl e!

Here is a brief di agramto show you how all the Hol y Days in a Roman year hi nge upon the day of Ti shri 1. The Hebrew ci vil year is reckoned from Tishri to Ti shri, so in that sense, two Hebrew years are i nvol ved when you specify all the Holy Days in one Roman year.

## How the Annual Hol ydays Are Governed By Ti shri 1

HEBREW YEAR HEBREW YEAR


How do you I ay the Hol y Days out on the Roman Cal endar? Si nce you det ermine Tishri l first, it's a good policy to work with the fall festivals first, then the Passover and Days of Unl eavened Bread, and finally Pent ecost. Of course, the day of the week for Pent ecost is al ways on a Sunday because of the way God defined the time for observing that Hol y Day. Neverthel ess, you still need to find the date for Pent ecost.

Take the "Sabbath pattern" as an illustration of the days of the week. Your reasoning will go something like this:

For this year, Trumpets (Ti shri 1) is on a SABBATH
At onement is 9 days later, or two days later in the week: MDNDAY Feast of Tabernacles begins on the 15th of Ti shri: SABBATH The Last Great Day is Tishri 22: SABBATH
(Or, in more abbrevi at ed fashi on, thi nk of Ti shri 1, 10, 15, 22 corresponding to Sabbath, Mbnday, Sabbath, Sabbath.)

Passover is 164 days bef ore Trumpets, or three days earlier in the week: WEDNESDAY

The first day of Unl eavened Bread is the next day: THURSDAY
The I ast day of Unl eavened Bread is Ni san 21, one week after Passover: WEDNESDAY
(I n a less verbose manner, thi nk Ni san 14 is 3 days earlier in the week than Trumpets. Then Ni san'14, 15, 21 correspond to Wednesday, Thur sday, Wednesday.)

Pentecost is a SUNDAY.

## Kossey

If you are still uncertain about how this pattern of days is thought out, please read Program 7 again. It will become much si mpl er once you understand the basic concepts of counting days that are expl ai ned in that program

Now practice on the four patterns yourself. You might want to do them more than once just to impress the thi nking process more fully in your mind. The four patterns of Holy Days are extremel y i mportant! Wbrk each pattern independently from the ot hers -- for your own benefit.
8. 1 What is the "Thursday pattern" of Hol y Days? In ot her words, what are the days of the week for each of the annual festivals when Trumpets is on a Thursday?
8. 2 What is the "Mbnday pattern" of Holy Days?
8. 3 What is the "Tuesday pattern" of Hol y Days?
8. 4 What is the "Sabbath pattern" of Hol y Days?

The answers are gi ven bel ow. (Don't try to memorize them UNLESS you can first figure them out yourself!)
8. 1 Ti shri 1 is on Thursday. Tishri $1,10,15,22$ correspond to

Thur sday, Sabbath, Thur sday, Thur sday.
Passover is 3 days earlier in the week than Trumpets: Mbnday.
Ni san 14, 15, 21 cor respond to Mbnday, Tuesday, Mbnday.
Pent ecost is on Sunday.
On an exam nation, of course, you should actually list these dates al ong with the Holy Day:

Passover, ( Ni san 14 ), Mbnday
1st day of Unl eavened Bread, ( Ni san 15), Tuesday
Last day of Unl eavened Bread, (Ni san 21), Mbnday
Pent ecost, Sunday Trumpets, (Tishri 1), Thur sday
At onement, (Tishri 10), Sabbath 1st day of Tabernacles, (Ti shri 15), Thur sday

Last Great Day, (Ti shri 22), Thur sday
8. 2 Ti shri 1 is on Mbnday. Ti shri 1, 10, 15, 22 correspond to Mbnday, Wednesday, Mbnday, Mbnday.
Passover is three days earlier in the week: Friday:
Ni san 14, 15, 21 cor respond to Friday, Sabbath, Fri day.
Pent ecost is on Sunday.
8. 3 Tishri 1 is on Tuesday. Tishri 1, 10, 15, 22 correspond to Tuesday, Thursday, Tuesday, Tuesday.
Passover is three days earlier in the week than Trumpets: Sabbath.
Ni san 14, 15, 21 correspond to Sabbath, Sunday, Sabbath.
Pentecost is on Sunday.
8. 4 See the exampl e gi ven previ ously in this programfor the answer.

Now that you can easily det er mine the day of the week for each of the annual Hol y Days, the final task is to specify the day of the month on a Roman cal endar for each Festival.

The Roman dates of the fall festivals are very easy to determine if you si mply work with the "seven pattern" of days in a month -- that you l earned in Program 7. All you are concerned with are the equi val ent Roman dates to Ti shri 1, 10, 15, and 22.

You will ei ther be gi ven the day of the week and the day of the Roman month for Trumpets, or el se you will cal culate the molad Ti shri for the requi red year yourself. The known correspondence bet ween the Hebrew cal endar and the Roman cal endar for one day provi des the key to det er min ng the other dates.

## Kossey

I magi ne a Hebrew cal endar for the month of Ti shri, but with most of the days except for $1,10,15$, and 22 del et ed.

## TI SHRI

1

| ( 8 ) | ( 9$)$ |  |
| :--- | :--- | :--- |
| 15 | 10 |  |
| 22 |  |  |

Suppose that Tishri 1 in a particul ar year occurred on Septenber 7. To find the dates of the other fall Holy Days, lay out an abbrevi at ed cal endar for the month of September (and Oct ober, if necessary):
$\begin{array}{rcc}\text { Ti shri } & 1 & \\ (8) & (9) & 10 \\ 15 & & \end{array}$
Sept enber: 7
15
22
(14) (15) 16

21
28
Essentially, all you are doing in either cal endar is counting by sevens from your starting day, the Feast of Trumpets. At onement is ni ne days later in the month, so it might hel pyou to think 1, 8-9-10, 15, 22 and correspondi ngl y for thi s exampl e, 7, 14-15-16, 21, 28.

As another illustration for the fall festivals, what happens when Trumpets is on September 25? Here you must remenber to change months, but that's no horrendous problem


Sept ember has 30 days, so you can si mpl y call Oct ober 1, September "31" for cal cul at i on purposes. Sept enber 46 is si mply 16 days after Sept enber 30, or Oct ober 16.

What happens when Trumpets is on August 28? The ot her fall Festivals will occur in the month of September. In order not to get i nvol ved wi th negative numbers, go ahead and add seven days, whi ch is equi val ent to finding Ti shri 8. What day is August 35? September 4, because August has 31 days. Sept ember 1 is the same as August 32; September 2 the same as August 33, et c.

Your abbrevi at ed cal endar will appear:

| Ti shri | 1 |  |  | August | 28 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $(8)$ | $(9)$ | 10 | Sept. | $(4)$ | (5) | 6 |
|  | 15 |  |  |  | 11 |  |  |
|  | 22 |  |  | 18 |  |  |  |

Observe that you are making a direct correspondence bet ween the Hebr ew cal endar and the Roman cal endar. Counting by full weeks -- "by sevens" -- enabl es you to avoi d ot her computational conf usi ons that can creep into your work if you just add days. You can add 9 days to the Roman date of Tishri 1 and find the date for At onement; 14 days to Trumpets for the first day of Tabernacles, and 21 days for the Last Great Day. Counting by sevens with an abbreviated cal endar is less error-prone, once you understand the principle.

Here are a few probl ems for you to practice determining the Roman dates for the annual Festivals. J ust i ndi cate the dates for At onement, the first day of Tabernacles, and the Last Great Day, but include the day of the week.
8. 5 Trumpet s is Sept enber 5, Mbnday
8. 6 Trumpet s i s Sept enber 12, Thursday

## Kossey

8. 7 Trumpets is Sept enber 23, Tuesday
9. 8 Trumpet s is August 25, Tuesday
10. 9 Trumpet s is Sept enber 16, Thur sday
11. 10 Trumpets is August 26, Sabbat h

The problems are worked for you on the next page.


The single quot ation mark dates are the answers, following the same format as above:
8. 6 Sept enber ' 12'

Thur sday ( Trumpet s)
, 23' (Oct 3) Thur sday
At onement
' 33' (Oct. 3) Thur sday
First day of Tabernacles
Last Great Day
8. 7 Sept ember ' 23 '

Tues day
$30-12 '$
1
7
' 14'

Oct ober
Oct ober
Oct ober
$2 \quad$ Thur sday

14 Tuesday
8. 8 August ' 25' Tuesday 32 - '34', or Sept enber $\begin{gathered}1 \\ , 1 \text { ' }\end{gathered}$
8. 9 Sept ember ' 16 '

Thur sday
Sabbath
, 23 - ' 25 '
' 37' whi ch is October '7'
Thur sday
Thur sday
8. 10 August ' 26 '

Sabbath
33-34, or September, 2 - ' 4 '
' 16'
Mbnday
Sabbat h
Sabbat h
Take a look at the spring season. Once you find the date for Passover (whi ch you'll l earn how to do very shortly), all you need to do is find the equi val ent dates for Ni san 15 and Ni san 21. An abbrevi at ed cal endar for Ni san looks like this:

| Ni san | 14 | 15 |
| :--- | :--- | :--- |
|  | 21 |  |

If Passover were April 18, you woul d have: April 18 19
The procedure is the same as for the fall season. Just remember that March has 31 days, so March 32 is the same as April 1. (You handl ed the 31 days in August the same way.)

Now indi cate the Roman dates and day of the week for the first day of Unl eavened Br ead and the I ast day of Unl eavened Br ead in the following probl ens:
8. 11 Passover is April 10, Mbnday
8. 12 Passover is March 25, Fri day
8. 13 Passover is March 29, Friday
8. 14 Passover is March 23, Wednesday

The probl ens are worked for yo on the next page.


Now turn your attention to finding the date of Passover, once you know the Roman date for Trumpets. As you learned earlier in this program Passover is 164 days bef ore the Feast of Trumpets. What you do is count by whol e Roman mont hs until you obtain a number si ightly I arger than 164. (You can al so find a number of days bel ow 164.)

An example will clarify the procedure. Suppose Trumpets is on Septenber 13. You want a number bi gger than 164 days, so you ment ally keep track of the mont hs you are adding:

| Sept ender : | 13 days |
| :---: | :---: |
| August: | 31 days |
| July: | 31 days |
| J une: | 30 days |
| May: | 31 days |
| April: | 30 days |
|  | 166 days |

166 days brings you to the last day of March. How many days past 164 did you go? $166-164=2$. Two days into April is April $2-$ Passover.

Doing this the ot her way, had you onl y counted as far as May, you woul d have come up with 136 days. 136 days brings you to April 30. How many days into April must you go? $164-136=28$ days. April 30 mi nus 28 days is April 2 -- Passover.

Take another ill ustration, Trumpets being on August 30. You have:

| August: | 30 days |
| :--- | ---: |
| July: | 31 days |
| June: | 30 days |
| May: | 31 days |
| April: | 30 days |
| March: | 31 days |
| ---183 days |  |

183 days takes you to the last day of February, How many days int o March is Passover? 183-164 = 19 days. Passover is March 19.

Once you practice on a few probl ens, you will have no difficulty in determing the Roman date for Passover. In these problens, al so i ndi cate the day of the week:
8. 15 Trumpets is Sept ember 8, Thur sday
8. 16 Tr umpets is Sept ember 11, Sabbat $h$
8. 17 Trumpets is Sept enber 5, Mbnday

## 8. 18 Trumpets is August 25, Sabbath

The answers are listed for you bel ow.
8. 15 Passover is March 28, Mbnday
8. 16 Passover is March 31, Wednesday
8. 17 Passover is March 25, Friday
8. 18 Passover is Mar ch 14, Wednesday

You' ve now l earned how to find the day of the week and the Roman day of the month for all the Hol y Days except Pentecost. Of course, you al ready know the day of the week. But how about the Roman dat e?

Bef ore you can count fifty days to Pentecost, you need the starting point, whi ch is the Sunday of the wavesheaf of fering. The table bel ow will gi ve you an overvi ew of the rel ati onship of Passover and the wavesheaf of fering:

| Passover (Ni san 14) | Wavesheaf of fering | Days after <br> Passover |
| :--- | :--- | :---: |
| Friday (Thurs. eve) | Ni san 16 (Sunday) | 2 |
| Wednesday | Ni san 18 | 4 |
| Mnday | Ni san 20 | 6 |
| Sabbath | Ni san 15 | 1 |

There's really no need to memorize this table, because you can easily determ ne the number of days from the Passover to the wavesheaf of fering once you find the day of the week Passover occurs. Just count from the Passover to the Sunday of the wavesheaf offering.

Wen the Passover is on Mbnday, March 28, the Sunday of the wavesheaf offering is 6 days later, March $34=$ April 3. With April 3 as the first day (counting incl usi vel y), Pentecost, the fiftieth day, will be 49 days later. There are $30-3$ days left in April, or 27 days. Pent ecost is 49 - 27 days into May, or on May 22. As another way for det er mining the date of Pent ecost, count seven full weeks from April Here you are counting by sevens: April 10, 17, 24, May 1, 8, 15, 22. Agai $n$ you have arrived at May 22 for the date of 'Pent ecost.

Here's another example for det erming the date of Pentecost. If Passover is on April 8, a Wednesday, the wavesheaf offering will be April 12, Sunday. Count seven full weeks from April 12: 19, 26, May 3, 10, 17, 24, 31. Pent ecost is May 31.

Wbrk out a few probl ens in order to firm up the process in your mind. Det ermine Pent ecost for each of the following cases
8. 19 Passover is Fri day, March 30
8. 20 Passover is Wednesday, March 16
8. 21 Passover is Sabbath, April 2
8. 22 Passover is Mbnday, April 20

The answers are on the next page.
8. 19 Pent ecost is Sunday, May 20
8. 20 Pentecost is Sunday, May 8
8. 21 Pentecost is Sunday, May 22

## 8. 22 Pent ecost is Sunday, J une 14

You have learned how to work with the Holy Days in separ ate steps fall festivals, Passover and Pentecost. Now integrate those skills and find for all the Hol y Days in a Roman year the respective date and day of the week.
8. 23 Trumpets is Mbnday, Sept enber 16
8. 24 Trumpets is Sabbath, Oct ober 3
8. 25 As the last problemin this series of prograns, determine the day of the week and the Roman date for all the annual festival s in the year 2055 AD. You may use the chart included in Program 4.

The answers are bel ow.
8. 23 Passover

1st day of Unl eavened Br ead 7th day of Unl eavened Br ead Pent ecost ( Tr umpet s At onement 1st day of Tabernacl es Last Great Day
8. 24

Passover
1st day of Unl eavened Br ead 7th day of Unl eavened Br ead Pent ecost

Friday, April 5
Sabbath, April 6
Friday, April 12
Sunday, May 26
Mbnday, Sept entber 16)
Wednesday, Sept ember 25
Mbnday, Sept entber 30
Mbnday, Oct ober 7
Wednesday, April 22
Thursday, April 23
Wednesday, April 29
Sunday, J une 14
( Tr umpet s
Sabbat h, Oct ober 3) At onement

Mbnday, Oct ober 12
1st day of Tabernacles Sabbath, Oct ober 17 Last Great Day

Sabbat h, Oct ober 24
8. 25 See Program 5, pages 44 and 45 for the determination of Tishri 1

Passover
1st day of Unl eavened Br ead 7th day of Unl eavened Br ead Pent ecost
Trumpet s At onement 1st day of Tabernacles Last Great Day

Mbnday, April 12
Tuesday, April 13
Mbnday, April 19
Sunday, J une 6
Thur sday, September 23
Sabbath, Oct ober 2
Thur sday, Oct ober 7
Thursday, Oct ober 14
\#\#\#------ - -

## SAMPLE TEST QUESTI ONS

1. How many lunar mont hs are in a 19-year cycle?
2. How many 19-year cycl es, I eap years, and common years are bet ween the bench mark and 171 AD?
3. W thout using your chart, determi ne the REDUCED number of days, hours, and parts in TWO common Hebrew years.
4. Demonstrate (or derive) without charts the amount that the Hebrew common year leads or trails the Roman cal endar.
5. What corrections (Roman leap year and Julian / Gregorian) are appl icable for the following years:

971 BC 1716 AD 1291 AD 1900 AD 2155 AD
6. In 1984 AD the mol ad Tishri is Septenber 25, Tuesday 11 h 976 p . What is the day of the week and the day of the month for Trumpets?
7. If the 29th of El ul is a Wednesday, what day of the week is the 15 th of El ul ?
8. What day of the week is 313 days bef ore a Friday?
9. In 1981 AD Trumpets is on September 28, Mbnday. Li st the day of the week and the day of the Roman month for each of the annual festivals gi ven in Leviticus 23.
10. Li st the day of the week and the day of the Roman month for all the annual festivals in the year 29 AD. You may use your chart.

## HEBREW CALENDAR

| Name of Mbnt h* | \#Sacred | \#Ci vil | Begi ns with new moon of |
| :---: | :---: | :---: | :---: |
| Avi v, or Ni san | 1st | 7t h | Mar ch- April |
| Zi w | 2nd | 8th | April-May |
| Si wan | 3 rd | 9th | May-J une |
| Tammz | 4t h | 10t h | J une- J ul y |
| Av | 5th | 11t h | Jul y-August |
| El ul | 6 th | 12t h | August - Sept ember |
| Ti shri, or Ethanim | 7th | 1st | Sept ember - Oct ober |
| Mar cheshwan | 8th | 2nd | Oct ober - Novenber |
| Ki sl ew | 9th | 3 rd | November - December |
| Tevet h | 10t h | 4 h | December - J anuar y |
| Shevat | 11 h | 5 h | J anuar y- Febr uar y |
| Adar | 12t h | 6 th | February- March |

(*) Mbder $n$ Hebrew transliteration. Pl ease compare with that bel ow. The spel I ing gi ven in W M Fel dman's, "Rabbi ni cal Mathematics and Astronomy," is as follows:

| Ni san | Ti shri |
| :--- | :--- |
| I yar | Marcheshvan |
| Si van | Kislev |
| Tammuz | Tebeth |
| Ab | Sh' bat |
| El ul | Adar |
|  | V' Adar |

Use this as a guide for pronunci ation.

## THE SI X TYPES OF HEBREW YEARS

| COMMDN |  |  |  |  | LEAP |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Mbnt h | Defici ent | Regul ar | Perfect | Deficient | Regul ar | Perfect |
| Ti shri | 30 | 30 | 30 | 30 | 30 | 30 |
| Marcheshwan <br> (Heshwan) | 29 | 29 | 30 | 29 | 29 | 30 |
| Ki sl ew | 29 | 30 | 30 | 29 | 30 | 30 |
| Teveth | 29 | 29 | 29 | 29 | 29 | 29 |
| Shevat | 30 | 30 | 30 | 30 | 30 | 30 |

Kossey

| Adar | 29 | 29 | 29 | 30 | 30 | 30 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| V' Adar |  |  |  | 29 | 29 | 29 |
| Ni san ( Avi v) | 30 | 30 | 30 | 30 | 30 | 30 |
| Zi w (lyar) | 29 | 29 | 29 | 29 | 29 | 29 |
| Si wan | 30 | 30 | 30 | 30 | 30 | 30 |
| Tammuz | 29 | 29 | 29 | 29 | 29 | 29 |
| Av | 30 | 30 | 30 | 30 | 30 | 30 |
| El ul | 29 | 29 | 29 | 29 | 29 | 29 |
|  | 353 | 354 | 355 | 383 | 384 | 385 |

(*) Mbdern Hebrew transliteration. See the top of this page for alternate spelling and pronunci ation.

